ges) **Reg. No. :** ....

de No. : 30566 E **Sub. Code : AMMA 21 / CMMA 21**

c. (CBCS) DEGREE EXAMINATION, APRIL 2022.

Second Semester

Mathematics — Core

FFERENTIAL EQUATIONS AND ANALYTICAL GEOMETRY OF THREE DIMENSIONS

(For those who joined in July 2020 onwards)

e : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

The complete solution of $y = px + p^2$ where $\left(p = \frac{dy}{dx}\right)$ is ————.

(a) $y = x^2 + c$ (b) $y = cx^2 - c$

(c) $y = cx + c^2$ (d) $cx - c$

2. The general solution of $(D^2 - 4)y = 0$ is $y =$ ————.

(a) $Ae^{2x} + Be^{-2x}$ (b) $Ae^{4x} + Be^{-4x}$

(c) $Ae^{3x} + Be^{x}$ (d) $Ae^{4x} + B$

3. The particular integral of $(D^2 - 9)y = \cos 3x$ is ————

(a) $\frac{\cos 3x}{18}$ (b) $\frac{\cos 3x}{9}$

(c) $\frac{\cos 3x}{-18}$ (d) 0

4. The solution of the differential equation $p^2 - 9p + 18 = 0$ where $p = \frac{dy}{dx}$ is ————

(a) $(y - 3x - c)(y - 6x - c) = 0$

(b) $(y - 6y - c)(y - 3x - c) = 0$

(c) $(x - 6y - c)(3x - y - c) = 0$

(d) $x^2 - 9x + 18 = 0$

5. The direction ratios of the line joining (1,2,–1) and (2, –1, 1) are ————.

(a) 2, 6, 4 (b) 1, –3, 2

(c) –2, –6, –4 (d) –1, 3, –2

The equation of the plane through (1,0,2) and parallel to the plane $2x + 3y - 4z = 0$ is ————

(a) $3x + 2y - 3z + 6 = 0$ (b) $3x + 2y - 3z - 6 = 0$

(c) $2x + 3y - 4z + 6 = 0$ (d) $2x + 3y - 4z - 6 = 0$

The line $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z-4}{-1}$ is parallel to the plane ————

(a) $x - 2y - 4z + 7 = 0$ (b) $2x - 2y - 4z + 7 = 0$

(c) $x - 7y - 4z + 7 = 0$ (d) $7x - 7y - 4z + 7 = 0$

If the line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ ———— the plane $ax + by + cz + d = 0$ then $al + bm + cn = 0$ and $ax_1 + by_1 + cz_1 + d = 0$.

(a) lies in (b) is parallel to

(c) bisects (d) is proportional to

Centre and radius of the sphere $x^2 + y^2 + z^2 - 6x - 2y - 4z - 11 = 0$ is

(a) (0,2,4) and 16 (b) (0,–1,2) and –4

(c) (3,1,2) and 5 (d) (1,–1,2) and –6

The equation of the tangent plane at the origin to the sphere $x^2+y^2+z^2-8x-6x+4z=0$ is ————

(a) $4x + 3y - 2z = 0$ (b) $4x - 3y - 2z = 0$

(c) $4x - 3y + 2z = 0$ (d) $-4x + 3y + 2z = 0$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve: $x^2(y - px) = yp^2$.

Or

(b) Solve: $p^2 + 2py\cot x - y^2 = 0$.

12. (a) Solve: $(D^3 - 3D^2 + 3D - 1)y = x^2 e^x$.

Or

(b) Solve: $x^2 y'' + 3xy' + y = \frac{1}{(1-x)^2}$.

13. (a) The line joining A(4,3,2) and B(1,2,–3) meets the planes YOZ, XOY in C,D respectively. Find the coordinates of C and and D and the ratios in which they divide AB.

Or

(b) Find the equation of the plane through the line of intersection of the plane $2x + y + 3z - 4 = 0$ and $4x - y + 5z - 7 = 0$ and perpendicular to the plane $x + 3y - 4z + 6 = 0$.

14. (a) Find the perpendicular distance form (3,9,–1) to the line $\frac{x+8}{-8}=\frac{y-31}{1}=\frac{z-13}{5}$

Or

(b) Find the equations of the plane passing through the line $5x-2y+7=0=x-3y+z-4$ and parallel to the line $\frac{x}{2}=\frac{y}{1}=\frac{z-1}{-2}$.

15. (a) Find the equation to the sphere through the four points (0,1,3), (1,2,4), (2,3,1) and (3,0,2)

Or

(b) Find the equation of the tangent line to the circle $x^2+y^2+z^2-x+4z=0$; $3x-2y+4z+1=0$ at the point (1,–2,–2).

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve: $\frac{dx}{dt}-\frac{dy}{dt}+x-y=1$

$2\frac{dx}{dt}+\frac{dy}{dt}=t$

Or

(b) Solve: $(px-y)(x+yp)=a^2p\ (x^2=x,\ y^2=y)$.

17. (a) Solve: $(D^2+1)y=x^2e^2x+x\cos x$.

Or

(b) Apply the method of variation of parameters to solve $y''=3y'=2y=x^2$

18. (a) A line makes an angles $\alpha,\beta,\gamma,\delta$ with the four diagonals of a cube. Prove that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$.

Or

(b) Show that the origin lies in the acute angle between the planes $x+2y+2z=9$, $4x-3y+12z+13=0$. Find the planes bisecting the angles between them and point out which bisects the obtuse angle.

19. (a) Find the equations of the image of the line $\frac{x-1}{2}=\frac{y+2}{-5}=\frac{z-3}{2}$ in the plane $2x-3y+2z+3=0$.

Or

(b) Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ are coplanar. Find their common point and find the equation of the plane which they lie.

20. (a) Find the equation of the sphere which passes through the circle $x^2+y^2+z^2-2x-4y=0$ $x+2y+3z=8$ and touches the plane $4x+3y=25$

Or

(b) Show that the conditions for the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ to cut the sphere $x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$ in a great circle is $2uu_1+2vv_1+2ww_1-(d+d_1)=2r_1^2$ where r_1 is the radius of the latter sphere.

(6 pages)

Reg. No. : ....

Code No. : 20291 E **Sub. Code : AMMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fourth Semester

Mathematics — Core

ABSTRACT ALGEBRA

(For those who joined in July 2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. Which of the following is not a symmetric relation on $S = \{a,b,c,d\}$?

   (a) $\{(a,b),(b,a)\}$

   (b) $\{(a,b),(b,c),(a,c)\}$

   (c) $\{(a,a),(b,b)\}$

   (d) $\{(a,b),(b,c),(b,a),(c,b)\}$

2. If the order of an element α is a group G is x then the order of the element α^{-1} is

   (a) -1 (b) $-x$

   (c) x (d) x^{-1}

3. In the group $G = \{1,-1,i,-i\}$ with usual multiplication, the inverse of i is ————

   (a) 1 (b) i

   (c) $-i$ (d) -1

4. Let G be a finite group and H be a subgroup of G. If $[G:H] = |G|$ then H is ————

   (a) $\{e\}$ (b) G

   (c) H (d) e

5. If $f: G \to G'$ is 1-1, then $O(\ker f) =$ ————

   (a) -1 (b) 0

   (c) 1 (d) 2

6. $R^{+}/\{1,-1\} \cong$ ————

   (a) R^{+} (b) R^{-}

   (c) R (d) $\{1,-1\}$

7. In the ring $(R,+,\cdot)$ the set of units is ————

   (a) Z (b) $\{1,-1\}$

   (c) $R-\{0\}$ (d) R

8. In the ring $M_2(R)$, the unit element is

   (a) $\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$ (b) $\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$

   (c) $\begin{pmatrix}2 & 4\\ 2 & 4\end{pmatrix}$ (d) $\begin{pmatrix}4 & 2\\ 2 & 4\end{pmatrix}$

9. Which one is a prime ideal in R?

   (a) (-1) (b) (0)

   (c) (1) (d) (2)

10. If $f(x), g(x) \in Z_4[x]$ be defined as $f(x) = x^2 + 3x + 1$ and $g(x) = 2x^2 + x$ then degree of $f(x).g(x)$ is ————

   (a) 3 (b) 4

   (c) 2 (d) 1

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Prove that the set of all equivalence classes determined by an equivalence relation defined on a set S forms a partition on the set S.

    Or

    (b) If $f: A \to B$, $g: B \to C$ are bijections, prove that $gof: A \to C$ is also a bijection.

12. (a) If G is a finite group with even number of elements then prove that G contains at least one element of order 2.

    Or

    (b) Let A and B be subgroups of a finite group G such that A is a subgroup of B. Show that $[G:A] = [G:B][B:A]$.

13. (a) Prove that every subgroup of a cyclic group is cyclic.

    Or

    (b) If $f: G \to G'$ is a group homomorphism prove that f is $1-1 \Leftrightarrow \ker f = \{e\}$.

[P.T.O.]

14. (a) Prove that a finite commutative ring R without zero - divisors is a field.

Or

(b) Show that the only ideals of a field F are F and $\{0\}$.

15. (a) Show that z_n is an integral domain if and only if n is prime.

Or

(b) Prove that every finite integral domain is a field.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 600 words.

16. (a) Let A and B be two subgroups of a group G. Prove that AB is a subgroup of G if and only if $AB = BA$.

Or

(b) Prove that the union of two subgroups of a group G is a subgroup if and only if one is contained in the other.

17. (a) Let H and K be two finite subgroups of a group G. Prove that $|HK| = \frac{|H||K|}{|H \cap K|}$.

Or

(b) State and prove Lagrange's theorem.

18. (a) If $f : G \to G'$ is a homomorphism with Kernel K, prove that $\frac{G}{K} \cong f(G)$.

Or

(b) State and prove Cayley's theorem.

19. (a) Let R be a commutative ring with identity prove that an ideal M of R is a maximal ideal $\Leftrightarrow R/M$ is a field.

Or

(b) Prove the following

(i) z_n is an integral domain $\Leftrightarrow n$ is a prime number.

(ii) the characteristics of an integral domain is either 0 or a prime number.

20. (a) State and prove division algorithm.

Or

(b) Prove that every integral domain can be embedded in a field.

(8 pages) **Reg. No. :** ....

Code No. : 20292 E **Sub. Code : AMMA 51**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fifth Semester

Mathematics – Core

LINEAR ALGEBRA

(For those who joined in July 2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. Which of the following is a subspace of a vector space R^3?
   (a) $W = \{(a, 0, 0)/a \in R\}$
   (b) $W = \{Ka, Kb, Kc/K \in R\}$
   (c) $W = \{(a, a+1, 0)/a \in R\}$
   (d) $W = \{(a, 0, b)/a, b \in R\}$

2. Let V be a vector space over a field F and W, a subspace of V. If $T: V \to \frac{V}{W}$ defined by $T(V) = W + V$ is a linear transformation, $\ker T =$ ———————.
   (a) $\{0\}$ (b) V
   (c) $\{1\}$ (d) W

3. If $S = \{(2, 0)\}$ in $V_2(R)$ then $L(S) =$ ———————.
   (a) $\{(x, 0)/x \in R\}$ (b) $\{(0, x)/x \in R\}$
   (c) $\{(0, 0)\}$ (d) $\{(0, 2)\}$

4. The vectors (a, b) and (c, d) are linearly dependent iff ———————.
   (a) $ab - cd = 0$ (b) $ac - db = 0$
   (c) $ab - bc = 0$ (d) $ad - bc = 0$

5. $T: V_2(R) \to V_2(R)$ given by $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ with respect to the standard basis then the linear transformation is ———————.
   (a) $T(a, b) = (a\sin\theta + b\cos\theta, -a\cos\theta + b\sin\theta)$
   (b) $T(a, b) = (a\cos\theta + b\sin\theta, -a\sin\theta + b\cos\theta)$
   (c) $T(a, b) = (-a\sin\theta + b\cos\theta, a\cos\theta + b\sin\theta)$
   (d) $T(a, b) = (-a\cos\theta + b\sin\theta, a\sin\theta + b\cos\theta)$

6. The norm of the vectors in $V_3(R)$ with standard inner product (3, –4, 0) is ———————.
   (a) 3 (b) 0
   (c) 5 (d) –4

7. The rank of the matrix is $\begin{bmatrix} 3 & -1 & 2 \\ 0 & 1 & -3 \\ 6 & -1 & 1 \end{bmatrix}$ is ———————.
   (a) 2 (b) 6
   (c) –2 (d) –3

8. If $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 2 & 1 \end{pmatrix}$ then $|A| =$ ———————.
   (a) 0 (b) 2
   (c) 4 (d) 1

9. For what value of k is 3 a characteristic root of $\begin{pmatrix} 3 & 1 & -1 \\ 3 & 5 & -k \\ 3 & k & -1 \end{pmatrix}$.
   (a) 5 (b) 2
   (c) –1 (d) 3

10. The eigen values of $\begin{pmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{pmatrix}$ are
    (a) 3, 4, 1 (b) 3, 5, 3
    (c) 3, 0, 0 (d) 1, 1, 2

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) (i) Prove that the intersection of two subspaces of a vector space is a subspace.

    (ii) Prove that the union of two subspaces of a vector space need not be a subspace.

    Or

    (b) Let V be a vector space over a field F. A non-empty subset W of V is a subspace of V iff $u, v \in W$ and $\alpha, \beta \in F \Rightarrow \alpha u + \beta v \in W$.

[P.T.O.]

12. (a) Prove that any subspace of a linearly independent set is linearly independent.

Or

(b) Prove that $S = \{(1,0,0),(0,1,0),(1,1,1)\}$ is a basis for $V_3(R)$.

13. (a) Let V be the set of all continuous real valued functions defined on the closed interval $[0,1]$. Prove that V is a real inner product space with inner product defined by

$$\langle f, g \rangle = \int_0^1 f(t)\, g(t)\, dt$$

Or

(b) Let V be a finite dimensional inner product space. Let W be a subspace of V. Prove that $(W^{\perp})^{\perp} = W$.

14. (a) Show that the non-singular matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ satisfies the equation $A^2 - 2A - 5I = 0$. Hence evaluate A^{-1}.

Or

(b) State and prove Cayley-Hamilton theorem.

15. (a) Let f be the bilinear form defined by $V_2(R)$ by $f(x, y) = x_1 y_1 + x_2 y_2$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Find the matrix of f w.r.t. the basis $\{(1,1),(1,2)\}$.

Or

(b) Find the characteristic roots of the matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let V and W be vector spaces over a field F. Let $L(V, W)$ represent the set of all linear transformations from V to W. Then $L(V, W)$ itself is a vector space over F under addition and scalar multiplication defined by $(f+g)(v) = f(v) + g(v)$ and $(\alpha f)(v) = \alpha f(v)$.

Or

(b) State and prove Fundamental theorem of Homomorphism.

17. (a) Let V be a finite dimensional vector space over a field F. Let W be a subspace of V. Prove that

(i) $\dim W \le \dim V$

(ii) $\dim \frac{V}{W} = \dim V - \dim W$.

Or

(b) Let V be a vector space over a field F. Let $S, T \subseteq V$, then prove that

(i) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$

(ii) $L(S \cup T) = L(S) + L(T)$

(iii) $L(S) = S \Leftrightarrow S$ is a subspace of V.

18. (a) Let V be the vector space of polynomials with inner product given by $\langle f, g \rangle = \int_0^1 f(t)\, g(t)\, dt$. Let $f(t) = t + 2$ and $g(t) = t^2 - 2t - 3$. Find

(i) $\langle f, g \rangle$

(ii) $\|f\|$.

Or

(b) Show that every finite dimensional inner product space has an orthonormal basis.

19. (a) Verify whether the following system of equations is consistent. If it is consistent find the solution

$$\begin{aligned} x - 4y - 3z &= -16 \\ 4x - y + 6z &= 16 \\ 2x + 7y + 12z &= 48 \\ 5x - 5y + 3z &= 0. \end{aligned}$$

Or

(b) Find the inverse of the matrix $\begin{pmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ using Cayley-Hamilton theorem.

20. (a) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.

Or

(b) Reduce the quadratic form $2x_1x_2 - x_1x_3 + x_1x_4 - x_2x_3 + x_2x_4 - 2x_3x_4$ to the diagonal form using Lagrange's method.

———————

(6 pages) **Reg. No. :** ....

Code No. : 20293 E **Sub. Code : AMMA 52**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fifth Semester

Mathematics — Core

REAL ANALYSIS

(For those who joined in July 2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. In [0, 1] with usual metric, $B\left(0, \frac{1}{4}\right)$ is ————.

(a) $\left(-\frac{1}{4}, \frac{1}{4}\right)$ (b) $\left[0, \frac{1}{4}\right]$

(c) $\left[0, \frac{1}{4}\right)$ (d) $\left(0, \frac{1}{4}\right)$

2. Which of the following subsets of R is not open?

(a) (0, 1) (b) ϕ

(c) $(1, 2) \cup (3, 4)$ (d) Q

3. $f : M_1 \to M_2$ is continuous if and only if

(a) $x_n - x = 0 \Rightarrow f(x_n) - f(x) = 0$

(b) $x_n \to x \Rightarrow f(x_n) = f(x)$

(c) $(x_n) \to x \Rightarrow (f(x_n)) \to f(x)$

(d) $x_n - x \to 0 \Rightarrow f(x_n - x) \to 0$

4. The function $f : (0, 1) \to R$ defined by $f(x) = \frac{1}{x}$ is

(a) not continuous

(b) uniformly continuous

(c) not uniformly continuous

(d) neither continuous nor uniformly continuous

5. If $A = (0, 1] \subseteq R$, then $\overline{A}$ is ————.

(a) (0, 1) (b) [0, 1]

(c) $(0, 1]$ (d) $[0,1)$

6. A connected subset of R is

(a) $[4, 7] \cup [8, 10]$ (b) $[4, 6] \cup [5, 7]$

(c) $[4, 7) \cup (7, 8)$ (d) Q

7. $\bigcup_{n=1}^{\infty} [0, n) = ?$

(a) $[0, \infty]$ (b) $(0, \infty)$

(c) $[0, \infty)$ (d) $(0, \infty]$

8. A compact subset of R is ————.

(a) $[0, \infty)$ (b) (3, 4)

(c) Q (d) [1, 2.8]

9. $\bigcup_{n=1}^{\infty} \left(0, \frac{1}{n}\right) = ?$

(a) (0,1) (b) ϕ

(c) {0} (d) (0, 1]

10. In $R \times R$, $\overline{Q \times Q}$ is ————.

(a) ϕ (b) Q^2

(c) $R \times R$ (d) $Z \times Z$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) In any metric space prove that each open ball is an open set.

Or

(b) Prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

12. (a) Show that the function $f : R \to R$ defined by

$$f(x) = \begin{cases} 0, & \text{if } x \text{ is irrational} \\ 1, & \text{if } x \text{ is rational} \end{cases}$$

is not continuous.

Or

(b) Prove that $f : M_1 \to M_2$ is continuous if and only if $f(\overline{A}) \subseteq \overline{f(A)}$ for all $A \subseteq M_1$.

13. (a) If A is a connected subset of the metric space M. Prove that $\overline{A}$ is connected.

Or

(b) Show that the continuous image of a connected metric space is connected.

14. (a) Prove that continuous image of a compact metric space is compact.

Or

(b) If A is a compact subset of a metric space (M,d), prove that A is closed.

[P.T.O.]

15. (a) Let A be a subset of a metric space M. If A is totally bounded, show that A is bounded.

Or

(b) Show that a metric space is compact if and only if any family of closed sets with finite intersection property has non empty intersection.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Cantor's interaction theorem.

Or

(b) State and prove Baire's category theorem.

17. (a) (i) Let (M, d) be a metric space. Let $a \in M$, show that the function $f : M \to R$ defined by $f(x) = d(x, a)$ is continuous.

(ii) Let (M, d) be any metric space. Let $f : M \to R$, $g : M \to R$ be two continuous functions. Prove that $f + g$ is continuous.

Or

(b) Prove that $f : R \to R$ is continuous at $a \in R$ f and only if $w(f, a) = 0$.

18. (a) Prove that R is a connected metric space.

Or

(b) (i) If A and B are connected subsets of a metric space M and $A \cap B = \phi$. Prove that $A \cup B$ is a connected set.

(ii) State and prove the Intermediate value theorem.

19. (a) State and prove Heine Borel Theorem.

Or

(b) Let (M_1, d_1) be a compact metric space and (M_2, d_2), be any metric space. If $f : M_1 \to M_2$ is continuous, prove that f is uniformly continuous on M.

20. (a) If A is a totally bounded set. Prove that $\overline{A}$ is also totally bounded.

Or

(b) Prove that the metric space M is compact iff any family $\{A_\alpha\}$ of closed sets with finite intersection property has non empty intersection.

———————

(8 pages)

Reg. No. : ....

Code No. : 20381 E **Sub. Code : CAMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

First/Third Semester

Mathematics — Allied

ALGEBRA AND DIFFERENTIAL EQUATION

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The n^{th} degree equation $f(x)=0$ cannot have more than ———— roots

(a) 4 (b) 6

(c) 7 (d) n

2. If α, β, γ are the roots of the equation $x^4 + px^3 + qx^2 + rx + 5 = 0$ then $\sum \alpha\beta\gamma =$ ————

(a) $-p$ (b) q

(c) $-r$ (d) s

3. After removing the fractional coefficients from the equation $x^3 - \frac{1}{4}x^2 + \frac{1}{3}x - 1 = 0$ we get ————

(a) $x^3 - 1 = 0$

(b) $12x^3 - 3x^2 + 4x - 12 = 0$

(c) $x^3 - 3x^2 + 48x - 1728 = 0$

(d) $x^3 - 3x^2 + 48x - 1 = 0$

4. How many imaginary roots will occur for the equation $x^7 - 3x^4 + 2x^3 - 1 = 0$?

(a) atmost four

(b) exactly four

(c) atleast four

(d) none of these

5. The characteristic equation of $\begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}$ is ————

(a) $\lambda^2 - 2\lambda - 1 = 0$

(b) $\lambda^2 + 2\lambda - 1 = 0$

(c) $\lambda^2 - 2\lambda + 1 = 0$

(d) $\lambda^2 + 2\lambda + 1 = 0$

6. Two eigen values of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ are –2 and 3, the third eigen value is ————.

(a) 4 (b) 5

(c) 6 (d) –1

7. The Clairauts equation is ————

(a) $y = cx + f(c)$

(b) $y = px + f(p)$

(c) $\frac{dy}{dx} = \left\{p + x\frac{dp}{dx}\right\} + f'(p)\frac{dp}{dx}$

(d) none of these

8. The partial differential equation obtained from $Z = ax + by + a^2$ by eliminating the arbitrary constants 'a' and 'b' is ————

(a) $Z = px + py + a^2$ (b) $Z = qx + py + a^2$

(c) $Z = px + qy + a^2$ (d) none of these

9. $L(x) =$ ————

(a) $\frac{1}{s}$ (b) $\frac{1}{s^2}$

(c) $-\frac{1}{s^2}$ (d) none of these

10. $L^{-1}\left[\frac{1}{s-a}\right] =$ ————

(a) 1 (b) x

(c) e^{ax} (d) e^{-ax}

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Solve $x^4 + 2x^2 - 16x + 77 = 0$ given that one of its root is $-2 = i\sqrt{7}$.

Or

(b) Solve the equation $81x^3 - 18x^2 - 36x + 8 = 0$ whose roots are in Harmonic progression.

12. (a) Diminish the roots of $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ by 2 and hence solve the original equation.

Or

(b) Solve the equation $x^3 - 4x^2 - 3x + 18 = 0$ given that two of its roots are equal.

13. (a) Find the eigen value and eigen vectors of $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

Or

(b) Find the inverse of matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$.

14. (a) Form the partial differential equation by eliminate arbitrary constants 'a' and 'b' from $\log(az - 1) = x + ay + b$.

Or

(b) Form a partial differential equation by eliminating arbitrary functions 'ϕ' from $\phi\left(x + y + z, x^2 + y^2 - z^2\right) = 0$.

15. (a) Find $L(\sin 2t \sin 3t)$.

Or

(b) (i) Prové that $L\left[e^{-ax}\right] = \dfrac{1}{s+a}$

(ii) If $L[f(x)] = F(s)$ then prove that $L[f(ax)] = \dfrac{1}{a} F\left(\dfrac{s}{a}\right)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Show that the roots of the equation $px^3 + qx^2 + rx + s = 0$ are in arithmetic progression if $2q^3 + 27p^2s = 9pqr$.

Or

(b) Solve $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$.

17. (a) Find by Horner's method, the positive root of $x^3 - 3x + 1 - 0$ lies between 1 and 2, Calculate it to three place of decimals.

Or

(b) Obtain by Newtons method, the root of the equation $x^3 - 3x + 1 = 0$ which lies between 1 and 2.

18. (a) Find the eigen value and eigen vectors of $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$.

Or

(b) Verify Cayley–Hamilton theorem for $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

19. (a) Solve $x^2 (y-z) p + y^2 (z-x) q = z^2 (x-y)$.

Or

(b) Solve $xp^2 - 2py + x = 0$.

20. (a) Find $L^{-1}\left[\dfrac{s^2 - s + 2}{s(s-3)(s+2)}\right]$.

Or

(b) Find $L^{-1}\left[\dfrac{cs + d}{(s+a)^2 + b^2}\right]$.

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Reg. No. : ....

ode No. : 30617 E **Sub. Code : CAMA 21**

CBCS) DEGREE EXAMINATION, APRIL 2022.

Second Semester

Mathematics — Allied

CTOR CALCULUS AND FOURIER SERIES

or those who joined in July 2021 onwards)

ne Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

hoose the correct answer :

. vector f is called solenoidal if ————.

) $\operatorname{div} \bar{f} = 0$ (b) $\operatorname{grad} \bar{f} = 0$

) $\operatorname{div} \bar{f} = 1$ (d) $\operatorname{curl} \bar{f} = 0$

f $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, then $\nabla \times \bar{r} =$ ————.

) $\bar{0}$ (b) 2

) 1 (d) $x^2 + y^2 + z^2$

3. If R is a rectangular region with vertices (0, 0), (2, 0), (2, 3) and (0, 3) then the value of $\iint_R dx\,dy =$ ————.

(a) 5 (b) 4

(c) 9 (d) 6

4. $\int_0^a \int_0^b \int_0^c dx\,dy\,dz =$ ————.

(a) $a+b+c$ (b) $a^3+b^3+c^3$

(c) abc (d) $a^3b^3c^3$

5. The value of $\int_0^{\pi}\int_0^1 r^4 \sin\theta\, dr\, d\theta$ is ————.

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) 1

6. If V is the volume enclosed by the closed surface S, then the value of $\iint_S \bar{r}\cdot\bar{n}\,dS =$ ————.

(a) $3V^2$ (b) $3V$

(c) $6V$ (d) 0

reen's theorem connects ————.

) line integral and double integral

) line integral and surface integral

) double integral and surface integral

) surface integral and volume integral

S is any closed surface enclosing a volume V nd $\bar{f} = ax\bar{i} + by\bar{j} + cz\bar{k}$, then $\iint_S \bar{f}\cdot\bar{n}\,dS =$ ————.

) $3V$ (b) $(a+b+c)V$

) $(a+b+c)^3 V^3$ (d) 0

$f(x)$ is an even function, ————.

) $f(x) = f(x^2)$ (b) $f(x) - f(x^2)$

) $f(x) = f(-x)$ (d) $f(x) = -f(-x)$

m is an integer, $\int_0^{\pi} \cos mx\,dx =$ ————.

) 1 (b) π

) $\frac{\pi}{2}$ (d) 0

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $\operatorname{curl}(\operatorname{curl}\bar{f}) = \operatorname{grad}\operatorname{div}\bar{f} - \nabla^2\bar{f}$.

Or

(b) Prove that $\operatorname{div}\left(\frac{\bar{r}}{r}\right) = \frac{2}{r}$.

12. (a) Find the area of the circle $x^2 + y^2 = r^2$ by using double integral.

Or

(b) Evaluate $\int_0^{\pi}\int_0^{a\cos\theta} r\sin\theta\, dr\, d\theta$.

13. (a) Find the work done by the force $\bar{F} = 3xy\bar{i} - 5z\bar{j} + 10x\bar{k}$ alone the curve C, $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$.

Or

(b) Evaluate $\iint_S \bar{f}\cdot\bar{n}\,dS$ where $\bar{f}=(x+y^2)\bar{i}-2x\bar{j}+2yz\bar{k}$ and S is the surface of the plane $2x+y+2z=6$ in the first octant.

14. (a) Verify Green's theorem for the function $\bar{f}=(x^2+y^2)\bar{i}-2xy\bar{j}$ and C is the rectangle in the xy plane bounded by $y=0$, $y=b$, $x=0$ and $x=a$.

Or

(b) Evaluate $\iint_S f.\bar{n}\,ds$ using Gauss divergence theorem for the vector function $\bar{f}=(x^3-yz)\bar{i}-2x^2y\bar{j}+2\bar{k}$ over the cube bounded by $x=0, y=0, z=0$, $x=a, y=a$ and $z=a$.

15. (a) Find half range cosines series for the function $f(x)=x^2$ in $(0,\pi)$

Or

(b) Express $f(x)=x$ $(-\pi<x<\pi)$ as a fourier series with period 2π.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) (i) If $\bar{r}$ is the position vector of any point $P(x,y,z)$, prove that grad $r^n = nr^{n-2}\bar{r}$.

(ii) Find the unit normal to the surface $x^3-xyz+z^3=1$ at (1,1,1)

Or

(b) (i) Prove that :
grad $(\varphi\psi)=\varphi grad\psi+\psi+grad\varphi$ and

(ii) grad $\left(\dfrac{\varphi}{\psi}\right)=\dfrac{\psi\, grad\varphi-\varphi grad\psi}{\psi^2}$.

17. (a) Evaluate $\iint_D x^2y^2\,dx\,dy$ where D is the circular disc $x^2+y^2\le 1$.

Or

(b) Evaluate the following :

(i) $\int_0^a\int_0^x\int_0^y xyz\,dz\,dy\,dx$

(ii) $\int_0^{\pi}\int_0^{\pi/2}\int_0^1 r^2\sin\theta\,dr\,d\theta\,d\varphi$

18. (a) If $\bar{f}=(2y+3)\bar{i}+xz\,j+(yz-x)\bar{k}$, evaluate $\int_C \bar{f}.\,d\bar{r}$ along the following paths C

(i) $x=2t^2$, $y=t$, $z=t^3$ from $t=0$ to $t=1$

(ii) The polygonal path P consisting of the three line segments AB, BC and CD where $A=(0,0,0), B=(0,0,1), C=(0,1,1)$ and $D=(2,1,1)$

(iii) The straight line joining $(0,0,0)$ and $(2,1,1)$.

Or

(b) If $\bar{f}=(x^2-y^2)\bar{i}+2xy\bar{j}$, evaluate $\int_C \bar{f}.\,d\bar{r}$ along the curve C is the xy- plane given by $y=x^2-x$ from the point $(1,0)$ to $(2,2)$.

19. (a) Verify Gauss divergence theorem for $\bar{f}=y\bar{i}+x\bar{j}+z^2\bar{k}$ for the cylindrical region S given by $x^2+y^2=a^2$, $z=0$ and $z=h$.

Or

(b) Verify Stoke's theorem for $\bar{f}=(2x-y)\bar{i}-yz^2-y^2z\bar{k}$ where S is the upper half of the sphere $x^2+y^2+z^2=1$ and C is its boundary.

20. (a) Find a cosine series in the range 0 to π for

$$f(x)=\begin{cases} x, & 0<x<\pi/2 \\ \pi-x, & \pi/2<x<\pi \end{cases}.$$

Or

(b) Expand $f(x)$ in $(-\pi,\pi)$ as a fourier series if

$$f(x)=\begin{cases} -x, & -\pi<x<0 \\ x, & 0\le x<\pi \end{cases}$$ and deduce

$$\frac{\pi^2}{8}=1+1/3^2+1/5^2+1/7^2+\ldots$$

(7 pages) **Reg. No. :** ....

Code No. : 20383 E **Sub. Code : CAMA 21**

B.Sc.(CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Second/Fourth Semester

Mathematics — Allied

VECTOR CALCULUS AND FOURIER SERIES

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The unit normal vector to the surface $x^3 - xyz^3 + z^3 = 1$ at $(1, 1, 1)$ is ————

(a) $\frac{2\vec{i} - \vec{j} + 2\vec{k}}{3}$ (b) $2\vec{i} - \vec{j} + 2\vec{k}$

(c) $\frac{\vec{i} - 2\vec{j} + 2\vec{k}}{3}$ (d) $\vec{i} + 2\vec{j} + 3\vec{k}$

2. If $\vec{F} = (x+3y)\vec{i} + (y-2z)\vec{j} + (x+az)\vec{k}$ solenoidal then the value of 'a' is ————

(a) 2 (b) −2

(c) 1 (d) 0

3. The value of $\int_0^1\int_0^1 (x^2+y^2)dxdy$ is ————

(a) $\frac{2}{3}$ (b) 2

(c) $\frac{1}{2}$ (d) 2

4. The value of $\int_0^a\int_0^a\int_0^a dzdydx$ is

(a) a^3 (b) a^2

(c) a (d) 1

5. $\int_0^{\pi/2} \left(3\sin x\vec{i} + 2\cos x\vec{j}\right) dx =$ ————

(a) $3\vec{i} + 2\vec{j}$ (b) $3\vec{i} - 2\vec{j}$

(c) $-3\vec{i} + 2\vec{j}$ (d) $-3\vec{i} - 2\vec{j}$

6. If S is the sphere $x^2+y^2+z^2=1$, the value of $\iint_S \vec{r}.\hat{n}ds$ is

(a) $\frac{4\pi}{3}$ (b) 3π

(c) 4π (d) 2π

7. If C is the circle $x=\cos\theta$, $y=\sin\theta$ then $\int_C (xdy-ydx)=$ ——————

(a) π (b) $\frac{\pi}{2}$

(c) 2π (d) $\frac{\pi}{4}$

8. $\iint_C \vec{F}.d\vec{r} = \iint_S (\nabla\times\vec{F})\hat{n}\,ds$ is

(a) Fundamental theorem

(b) Gauss-divergence theorem

(c) Green's theorem

(d) Stoke's theorem

9. If $f(x)=\begin{cases}1, & 0<x<\pi\\ 0, & \pi<x<2\pi\end{cases}$ then the value of the Fourier coefficinet a_n is

(a) 0 (b) –2

(c) –3 (d) –4

10. If $f(x)=|x|$ in $(-\pi,\pi)$ then the fourier coefficient a_0 is

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{3\pi}{2}$ (d) $\frac{\pi^2}{2}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Obtain the directional derivative of $\phi = xy^2+yz^3$ at the point $(2,-1,1)$ in the direction of $\vec{i}+2\vec{j}+2\vec{k}$

Or

(b) Prove that curl grad $\phi = \nabla\times\nabla\phi = 0$.

[P.T.O.]

12. (a) Evaluate $\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2+y^2}$ using change of order of integration.

Or

(b) Evaluate $\int_0^{\log a} \int_0^x \int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$

13. (a) Evaluate $\int_C \vec{f}.d\vec{r}$ where $\vec{f} = (x^2+y^2)\vec{i} + (x^2-y^2)\vec{j}$ and C is the curve $y = x^2$ joining (0, 0) and (1,1)

Or

(b) Evaluate $\vec{f} = y^2\vec{i} + y\vec{j} - xz\vec{k}$ for a vector S $x^2+y^2+z^2=a^2$ in the upper hemisphere and $z \geq 0$, $\iint_S (\nabla \times \vec{F})\hat{n}\,ds$.

14. (a) Use Green's theorem to evaluate $\int_C (x^2y\,dx + y^3\,dy)$. where C is the closed path formed by $y = x$ and $y = x^3$ from (0,0) to (1,1).

Or

(b) Evaluate $\int_C (e^x dx + 2y\,dy - dz)$, by using stoke's theorem where C is the curve $x^2+y^2=4$, $z=2$.

15. (a) Show that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$ in the interval $-\pi \leq x \leq \pi$

Or

(b) Find a sine series for $f(x) = c$ in the range 0 to π

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Prove that $\nabla \times (\nabla \times \vec{f}) = \nabla(\nabla.\vec{f}) - \nabla^2 f$

Or

(b) If $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ is irrotational. If $\vec{F} = \nabla\phi$ then find the value of ϕ.

17. (a) Find the area of the region D bounded by the parabolas $y = x^2$ and $x = y^2$.

Or

(b) Evaluate $\iint_D x^2y^2\,dx\,dy$, where D is the circular disc $x^2+y^2 \leq 1$.

18. (a) Find the work done by the force $\vec{F} = 3xy\bar{i} - 5z\bar{j} + 10x\bar{k}$ along the curve $x = t^2 + 1,\ y = 2t^2,\ z = t^3$ from $t = 1$ to $t = 2$.

Or

(b) Evaluate $\iiint xyzdxdydz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$

19. (a) Verify Gauss theorem for $\vec{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ taken over the cuboid $0 \le x \le a$, $0 \le y \le b$ and $0 \le z \le c$

Or

(b) Verify stoke's theorem for $\vec{F} = (x^2 - y^2)\bar{i} + 2xy\bar{j}$ int he rectangular region $x = 0$, $x = a$, $y = 0$, $y = b$.

20. (a) Express $f(x) = \frac{1}{2}(\pi - x)$ as a fourier series with period 2π, to be valid in the interval $(0, 2\pi)$

Or

(b) Find a cosine series in the range 0 to π for

$$f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$$

(8 pages) Reg. No. : ....

Code No. : 20382 E **Sub. Code : CAST 11**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

First/Third Semester

Mathematics – Allied

STATISTICS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The value of μ_3' is ————

(a) $\mu_3 + 3\mu_2\mu_1' + \mu_1'^3$

(b) $\mu_3 - 3\mu_2\mu_1' + \mu_1'^3$

(c) $\mu_3 + 2\mu_2\mu_1' + \mu_1'^3$

(d) $\mu_3 - 2\mu_2\mu_1' + \mu_1'^3$

2. ———— is not affected by change of origin but affected by change of scale.

(a) arithmetic mean

(b) median

(c) moment

(d) mode

3. If $\sum(x-\bar{x})^2 = 60$, $\sum(y-\bar{y})^2 = 90$, $\sum(x-\bar{x})(y-\bar{y}) = 45$, then the correlation coefficient between the variables x and y is ————

(a) 0.6125 (b) 0.1265

(c) 0.5623 (d) 0.2516

4. If X and Y are uncorrelated, $\operatorname{cov}(X,Y) =$ ————

(a) 0 (b) 1

(c) –1 (d) ∞

5. If $Q = 0$, then $Y =$ ————

(a) 1 (b) 0

(c) –1 (d) $-\infty$

6. For any given three attributes, the total number of positive class frequencies is ————

(a) n^2 (b) n^3

(c) 3^n (d) 2^n

7. The value for c for the probability density function $f(x)=\frac{x}{c}$, $x=1,2,3,4,5$ is ————

(a) $\frac{1}{15}$ (b) 15

(c) $\frac{15}{2}$ (d) 1

8. If X is the number on a die when it is thrown, $E(X)=$ ————

(a) $\frac{1}{6}$ (b) 1

(c) 7 (d) $\frac{7}{2}$

9. If the mean of a Poisson distribution is λ, standard deviation = ————

(a) λ (b) λ^2

(c) $\sqrt{\lambda}$ (d) $\sqrt{\lambda^2+\lambda}$

10. In a normal distribution, Q.D = ———— S.D.

(a) $\frac{4}{5}$ (b) $\frac{2}{3}$

(c) $\frac{3}{2}$ (d) $\frac{5}{4}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) In a frequency distribution, Bowley's coefficient of skewness is 0.6, sum of the upper and lower quartiles is 100, median is 38. Find the value of the upper quartile.

Or

(b) Fit a straight line to the following data :

X	1	2	3	4	6	8
Y	2.4	3	3.6	4	5	6

12. (a) Prove that $-1 \le \gamma \le 1$.

Or

(b) From the following table, find the rank correlation coefficient between the height and weight.

Height (in cm)	165	167	166	170	169	172
Weight (in kg)	61	60	63.5	63	61.5	64

13. (a) Is there any inconsistency in the data given below $N = 600$; $(A) = 300$; $(B) = 400$; $(AB) = 50$.

Or

(b) Show that the relation between Yule's coefficient Q and the coefficient of colligation Y is $Q = \frac{2Y}{1+Y^2}$.

14. (a) If $f(x) = \begin{cases} Ax & for\ 0 < x < 5 \\ A(10-x) & for\ 5 \le x < 10 \\ 0 & otherwise \end{cases}$, is the probability density function of a random variable X, find the value of A.

Or

(b) If the random variable X has the following probability law $P(X = x) = q^{x-1} \cdot p$, $x = 1, 2, 3, \ldots$, find the moment generating function of X.

15. (a) If the mean of a normal distribution is 4 and the variance is 3, find its mode.

Or

(b) A book of 500 pages contains 500 mistakes. Find the probability that there are atleast four mistakes in a randomly selected page.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Calculate the first three central moments for the following data :

x	3	6	10	15	20	23	24
f	2	5	15	21	16	13	4

Or

(b) Fit a curve $y = ae^{bx}$ for the following data :

x	0	1	2	3
y	3	8	25	74

17. (a) Find the correlation coefficient from the following data :

x	65	66	67	67	68	69	70	71
y	67	68	65	68	72	72	69	71

Or

(b) Let x, y be two variables with standard deviations σ_x and σ_y respectively. If $u = x + ky$, $v = x + \left(\frac{\sigma_x}{\sigma_y}\right) y$ and $\gamma_{uv} = 0$, find the value of k.

18. (a) If $(A)=50$, $(B)=60$, $(C)=80$, $(AB)=35$, $(AC)=45$ and $(BC)=42$, find the greatest and least value of (ABC).

Or

(b) Show that for n attributes $A_1, A_2, \ldots, A_n$, $(A_1A_2\ldots A_n) \geq (A_1)+(A_2)+\ldots+(A_n)-(n-1)N$.

19. (a) If x and y are two random variables, determine whether they are independent in the following cases.

(i) $f(x,y)=\begin{cases} 8xy; 0<x<1, 0<y<x \\ 0 \quad otherwise \end{cases}$

(ii) $f(x,y)=\begin{cases} 4xy; 0<x<1, 0<y<1 \\ 0 \quad otherwise \end{cases}$

Or

(b) State and prove the addition and multiplication theorems of expectation for continuous random variables.

20. (a) If X is a Poisson variate such that $P(X=2)=9\ P(X=4)+90P(X=6)$, find the value of β.

Or

(b) For a normal distribution, prove that $\mu_{2n}=(2n-1)\sigma^2\mu_{2n-2}$.

(7 pages)

Reg. No. : ....

Code No. : 20378 E **Sub. Code : CMMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

First Semester

Mathematics – Core

CALCULUS AND CLASSICAL ALGEBRA

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The radius of curvature for the curve $y = e^x$ at the point where it crosses the y-axis is ________.

   (a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$

   (c) 2 (d) $2\sqrt{2}$

2. If $x = f(t)$ and $y = g(t)$, then the radius of curvature is ________.

   (a) $\frac{\left(x_1^2 + y_1^2\right)^{3/2}}{x_1 y_1 + x_2 y_1}$ (b) $\frac{\left(x_1^2 + y_1^2\right)^{3/2}}{x_2 y_2 + x_1 y_1}$

   (c) $\frac{\left(x_1^2 - y_1^2\right)^{3/2}}{x_1 y_2 - x_2 y_1}$ (d) None of these

3. In Polar co-ordinates $\frac{\partial(x, y)}{\partial(r, \theta)} =$ ________.

   (a) x (b) θ

   (c) r (d) $r\theta$

4. $\int_0^1 \int_0^1 xy \, dx \, dy =$ ________.

   (a) $\frac{1}{2}$ (b) $\frac{1}{4}$

   (c) $\frac{1}{3}$ (d) None of these

5. $\left\lceil\left(\frac{1}{2}\right) =$ ________

   (a) $\frac{\sqrt{\pi}}{2}$ (b) $\sqrt{\pi}$

   (c) π (d) $\frac{\sqrt{\pi}}{3}$

6. Transformations of Beta function $\beta\ (m,n)$ = ____________

(a) $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ (b) $\int_1^0 x^{m-1}(1-x)^{n-1}dx$

(c) $\int_0^\infty x^{m-1}(1-x)^{n-1}dx$ (d) None of these

7. The n[th] degree equation $f(x) = 0$ can't have more than ____________ roots.

(a) 4 (b) 6

(c) 7 (d) n

8. If α, β, γ are the roots of the equation $x^3 - 2x^2 + 5x - 7 = 0$ then $\alpha\beta + \beta\gamma + \gamma\alpha$ = ____________.

(a) 2 (b) 5

(c) –5 (d) 7

9. By removing the fractional co-efficients from $x^3 - \frac{1}{4}x^2 + \frac{1}{3}x - 1 = 0$ multiply the roots by ____________.

(a) 4 (b) 3

(c) –3 (d) 12

10. The equation $z^3 - 3z + 1 = 0$ has a real root between ____________.

(a) 1,2 (b) 2,3

(c) 3,4 (d) 4,5

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the radius of curvature of the cardioid $r = a(1 - \cos\theta)$.

Or

(b) For the curve $x^3 + y^3 = 3axy$, show that the radius of curvature is $\frac{3\sqrt{2}\,a}{16}$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$.

12. (a) If $x + y + z = u,\ y + z = uv,\ z = uvw$ prove $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$.

Or

(b) Evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$.

13. (a) Prove that $\int_0^{\frac{\pi}{2}} \sin^7\theta \cos^5\theta \, d\theta = \frac{1}{120}$.

Or

(b) Simplify $\int_0^{\frac{\pi}{2}} \sin^{10}\theta \, d\theta$.

14. (a) If α, β, γ are the roots of the equation $x^3 + px^2 + r = 0$ find the value of $\alpha^3 + \beta^3 + \gamma^3$.

Or

(b) Frame an equation with rational co-efficient, one of whose roots is $\sqrt{5} + \sqrt{2}$.

15. (a) Solve $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$

Or

(b) Transform the equation $x^4 + x^3 - 3x^2 + 2x - 4 = 0$ whose roots are each diminished by 2.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find 'P' at the point 't' of the curve $x = a(\cos t + t\sin t), \; y = a(\sin t - t\cos t)$

Or

(b) Find the equation of the evolute of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

17. (a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dx \, dy \, dz$.

Or

(b) Change the order of integration and evaluate $\int_0^b \int_0^{\frac{a}{b}\sqrt{b^2-y^2}} xy \, dx \, dy$.

18. (a) Show that $\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma(m+n)}$

Or

(b) Express $\int_0^1 x^m (1-x^n)^p dx$ interms of Gamma function and evaluate $\int_0^1 x^5 (1-x^3)^{10} dx$.

19. (a) Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ are in G.P.

Or

(b) Find correct to two place of decimals the root of the equation $x^4 - 3x + 1$ that lies between 1 and 2 by Newton method.

20. (a) Solve $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$

Or

(b) Discuss the nature of the roots of the equation $x^5 - 6x^2 - 4x + 5 = 0$.

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Reg. No. : ....

Code No. : 20379 E **Sub. Code : CMMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Second Semester

Mathematics — Core

DIFFERENTIAL EQUATIONS AND ANALYTICAL GEOMETRY OF THREE DIMENSIONS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer:

1. Let m be the order of a given differential equation then

(a) m is any integer

(b) m is any real number

(c) m is any positive integer

(d) None of the above

2. What is the degree of the differential equation $3y = x\frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right)$?

(a) 1

(b) 2

(c) 8

(d) None of the above

3. The roots of the auxiliary equation of the differential equation $(D^2 + 3D + 2)y = 0$ is ____________

(a) 1, 2

(b) $-1, -2$

(c) $-1, 2$

(d) $1, -2$

4. The roots of the auxiliary equation of the differential equation $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = 0$ is ____________

(a) 1, 1

(b) $1 \pm i$

(c) $-1, -1$

(d) $1, -1$

5. If a_1, b_1, c_1 and a_2, b_2, c_2 are the direction ratios of two perpendicular lines then $a_1a_2 + b_1b_2 + c_1c_2 =$ ____________

(a) 1

(b) −1

(c) 0

(d) None of the above

6. Angle between two diagonals of a cube is ____________

(a) $\cos(1/3)$ (b) $\sin(1/3)$

(c) $\cos^{-1}(1/3)$ (d) $\sin^{-1}(1/3)$

7. If the line is parallel to the plane then $\sin\theta =$ ____________

(a) 0

(b) 1

(c) $\frac{1}{\sqrt{2}}$

(d) None of the above

8. If the shortest distance in zero the lines are ____________

(a) coplanar (b) non coplanar

(c) skewlines (d) none

9. Radius of the sphere $x^2 + y^2 + z^2 - 2x - 4y - bz - 2 = 0$ is ____________

(a) 2 (b) 3

(c) 4 (d) 5

10. The condition for orthogonality of two sphere is ____________

(a) $2uu' + 2vv' + 2ww' = d + d'$

(b) $uu' + vv' + ww' = d + d'$

(c) $2uu' + 2vv' + 2ww' = d - d'$

(d) $uu' + vv' + ww' = d - d'$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Solve $xp^2 - 2yp + x = 0$

Or

(b) Solve $y = xp + x(1 + p^2)^{1/2}$

12. (a) Solve $(D^2 + 4)y = x \sin x$

Or

(b) Solve $x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$

13. (a) If the line whose direction cosines are given by $al + bm + cn = 0$ and $mn + nl + lm = 0$ are perpendicular, prove $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.

Or

(b) Find the angle between $13x + 6y + 5z + 1 = 0$ and $6z - 4y - 2x + 81 = 0$

14. (a) Find the equation of the straight line through $(1, 0, 2)$ and parallel to the planes $2x + 3y - z = 1$ and $2x + y + z = 7$.

Or

(b) Find the angle between the line $\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-1}{6}$ and plane $3x + y + z = 1$.

15. (a) Find the equation of the sphere which passes through the points $(3, 4, 2)$, $(2, 0, 5)$, $(2, 4, 5)$, $(3, 3, 1)$.

Or

(b) Show that the plane $2x + y - 2z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x + 2y - 4z - 3 = 0$. find the point of contact.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Solve $Z = px + qy + p^2q^2$

Or

(b) Solve $\frac{dx}{dt} + 2x - 3y = t;\ \frac{dy}{dt} - 3x + 2y = e^{2t}$

17. (a) Solve $(D^2 + 1)y = x^2e^{2x} + x\cos x$

Or

(b) Solve

$$(5 + 2x)^2 \frac{d^2y}{dx^2} - 6(5 + 2x)\frac{dy}{dx} + 8y = 6x$$

18. (a) Find the equation of the plane passing through the three points $(2, 3, 4)$, $(-3, 5, 1)$ and $(4, -1, 2)$.

Or

(b) Find the equation of the plane through the line of intersection of the planes $3x + 2y + 3 = 0$, $2x + y - z + 2 = 0$ and parallel to $x + y + z = 2$.

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Code No. : 20380 E **Sub. Code : CMMA 31**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Third Semester

Mathematics — Core

SEQUENCES AND SERIES

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The following statements are true except

(a) $\left(\frac{1}{n}\right)$ is a convergent sequence

(b) $\left(\frac{1}{n}\right)$ is a bounded sequence

(c) $\left(\frac{1}{n}\right)$ is a monotonic increasing sequence

(d) $\left(\frac{1}{n}\right)$ is a strictly mono

2. Read the following statements

(i) Any convergent sequence is a Cauchy sequence

(ii) Any Cauchy sequence is a convergent sequence

(iii) Any Cauchy sequence is a bounded sequence

(iv) Any bounded sequence is a Cauchy sequence

The correct statement

(a) only (i) and (iii) are true

(b) only (ii) and (iv) are true

(c) (i), (ii), (iii) and (iv) are true

(d) only (i) is true

3. The incorrect statement from the following (K_1, K_2)

(a) $1+2+3+4+\cdots$ diverges to ∞

(b) $\sum_1^\infty\left(\frac{1}{2^n}\right)$ converges to 1

(c) $\sum_1^\infty\left(\frac{1}{3^n}\right)$ converges to $\frac{1}{2}$

(d) $\sum_1^\infty\left(\frac{1}{n}\right)$ converges to 2

4. (i) The series $\sum_{n=1}^\infty \frac{1}{n^p}$ converges if $p<1$

(ii) The series $\sum_{n=1}^\infty \frac{1}{n^p}$ converges if $p>1$

The correct statement is ————

(a) only (i) is false

(b) only (ii) is false

(c) both (i) and (ii) are false

(d) both (i) and (ii) are true

5. $1+\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\cdots=$ ————

(a) 2 (b) –2

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

6. $\underset{n\to\infty}{lt}\frac{1}{n}\left(1+\frac{1}{2}+\cdots+\frac{1}{n}\right)=$ ————

(a) 0 (b) e

(c) 1 (d) None

7. Let Σa_n be a series of positive terms. The correct statement from the following is

(a) Σa_n converges if $\underset{n\to\infty}{lt}\frac{a_n}{a_{n+1}}>1$

(b) Σa_n converges if $\underset{n\to\infty}{lt}\frac{a_n}{a_{n+1}}<1$

(c) Σa_n converges if $\underset{n\to\infty}{lt}\frac{a_{n+1}}{a_n}>1$

(d) Σa_n converges if $\underset{n\to\infty}{lt}\frac{a_{n+1}}{a_n}>0$

8. Applying the ratio test for $1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots+\frac{1}{n!}+\cdots$ the series is

(a) convergent

(b) divergent

(c) neither convergent nor divergent

(d) both convergent and divergent

9. $\underset{n\to\infty}{lt}\left(1+\frac{1}{1!}+\frac{1}{2!}+\cdots+\frac{1}{n!}\right)=$ ————

(a) 0 (b) 1

(c) e (d) ∞

10. $\lim_{n\to\infty} \frac{(1^3+2^3+\cdots+n^3)}{n^4} =$ ________

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{1}{4}$ (d) 0

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Show that a sequence cannot converge to two different limits.

Or

(b) Prove that if Σa_n converges and Σb_n diverges then $\Sigma(a_n+b_n)$ diverges.

12. (a) If $(a_n)\to a$ and $(b_n)\to b$ prove that $(a_n b_n)\to ab$.

Or

(b) Test the convergence of the Geometric series $1+r+r^2+\cdots+r^n+\cdots$ when

(i) $0\le r\le 1$

(ii) $r>1$

(iii) $r=1$.

13. (a) Discuss the convergence of the series $\sum\frac{1}{\sqrt{n^3+1}}$.

Or

(b) If $y=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$ prove that $x=\frac{y}{1!}+\frac{y^2}{2!}+\frac{y^3}{3!}+\cdots$.

14. (a) Test the convergence of $\Sigma\frac{n^n}{n!}$.

Or

(b) Test the convergence of $\sum\sqrt{\frac{n}{n+1}}.x^n$.

15. (a) Test the convergence of $\Sigma\frac{(-1)^n \sin n\alpha}{n^3}$.

Or

(b) State and prove Dirichlet's test.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 600 words.

16. (a) Show that the sequence $\left(1+\frac{1}{n}\right)^n$ converges.

Or

(b) If $(a_n)\to a$ and $a_n\ne 0$ for all n and $a\ne 0$ then prove that $\left(\frac{1}{a_n}\right)\to\frac{1}{a}$. Also prove $\left(\frac{a_n}{b_n}\right)\to\frac{a}{b}$ if $(a_n)\to a, (b_n)\to b$ where $b_n\ne 0$ for all n and $b\ne 0$.

17. (a) Applying Cauchy's general principle of convergence prove that $1-\frac{1}{2}+\frac{1}{3}-\cdots+(-1)^n\frac{1}{n}+\cdots$ is convergent.

Or

(b) Show that the harmonic series $\Sigma\frac{1}{n^p}$ converges if $p>1$ and diverges if $p\le 1$.

18. (a) State and prove comparison test.

Or

(b) State and prove Kummer's test.

19. (a) Test the convergence of the series $1+\frac{\alpha\beta}{r}x+\frac{\alpha(\alpha+1)\beta(\beta+1)}{r(r+1)2!}x^2+\cdots$.

Or

(b) Test the convergence and divergence of the series $1+\frac{2x}{2!}+\frac{3^2x^2}{3!}+\frac{4^3x^3}{4!}+\frac{5^4x^4}{5!}+\cdots$.

20. (a) State and prove Cauchy's condensation test.

Or

(b) Test the convergence of the series $\sum(-1)^n\left(\sqrt{n^2+1}-n\right)$.

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Code No. : 20387 E **Sub. Code : CNMA 31**

U.G. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Third Semester

Mathematics

Non Major Elective — MATHEMATICS FOR COMPETITIVE EXAMINATIONS — I

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. $7+7\div7\times7=$ ————

(a) $\frac{2}{7}$ (b) 14

(c) $7\frac{1}{7}$ (d) 42

2. $5005-5000\div10.00=$ ————

(a) 0.5 (b) 50

(c) 5000 (d) 4505

3. The average of first five multiples of 3 is

(a) 9 (b) 72.6

(c) 3 (d) 6

4. If $2A=3B=4C$ then $A:B:C$ is ————

(a) 2 : 3 : 4 (b) 4 : 3 : 2

(c) 6 : 4 : 3 (d) 3 : 4 : 6

5. ———— % of 64 is 8.

(a) 3 (b) 10

(c) 8 (d) 12.5

6. The number increased by $37\frac{1}{2}\%$ gives 33. The number is ————

(a) 22 (b) 24

(c) 25 (d) 27

7. A man sold a ratio for Rs. 1980 and gained 10% the ratio was bought for ————

(a) Rs. 1782 (b) Rs. 1800

(c) Rs. 2178 (d) Rs. 1500

8. Cost price = Rs. 56.25. profit = 20% selling price = ————

(a) Rs. 62.50 (b) Rs. 60

(c) Rs. 67.50 (d) Rs. 66.25

9. The difference of two numbers is 8 and $\frac{1}{8}$th of their sum is 35. The numbers are ————

(a) 132, 140 (b) 128, 136

(c) 124, 132 (d) 136, 144

10. 11 times a number gives 132. The number is ————

(a) 11 (b) 12

(c) 13.2 (d) 13

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Find the value of $1+\cfrac{1}{2+\cfrac{1}{1-\cfrac{1}{3}}}$.

Or

(b) Find the value of $\frac{9^2\times18^4}{3^{16}}$.

12. (a) The average age of a family of 6 members is 22 years. If the age of the youngest member be 7 years, find the average age of the family at the birth of the youngest member.

Or

(b) In a mixture of 35 litres, the ratio of milk and water is 4 : 1 now, 7 litres of water is added to the mixture. Find the ratio of milk and water in the new mixture.

13. (a) A person 'A' credits 15% of his salary in his fixed deposit account and spends 30% of the remaining amount on groceries. If the cash in hand is Rs. 2380, what is his salary?

Or

(b) A and B started a business and invested Rs. 20,000 and Rs. 25,000 respectively. After 4 months B left and C joined by investing Rs. 15,000. At the end of the year, there was a profit of Rs. 4,600. What is the share of C?

14. (a) By selling a watch for Rs. 144, a man loses 10%. At what price should he sell it to gain 10%?

Or

(b) A man sells an article at a profit of 20%. If he had bought it at 20% less and sold if for Rs. 5 less, he would have gained 25%. Find the cost price of the article.

15. (a) A fraction becomes 4 when 1 is added to both numerator and denominator and it becomes 7 when 1 is subtracted from both the numerator and denominator. What is the numerator of the fraction?

Or

(b) The number x is exactly divisible by 5 and the remainder obtained on dividing the number y by 5 is 1. What remainder will be obtained when $x+y$ is divided by 5?

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Evaluate $\frac{0.125+0.027}{0.25-0.15+0.09}$.

Or

(b) The average of 5 consecutive numbers is n. If the next two numbers are also included, the average will be increased by how much?

17. (a) Two numbers are in the ratio 3 : 5. If each number is increased by 10, the ratio becomes 5 : 7. Find the numbers.

Or

(b) The ratio of milk and water in 85 Kg of adulterated milk is 27 : 7. Find the amount of water which must be added to make the ratio 3 : 1.

18. (a) A and B invest in the business in the ratio 3 : 2. If 5% of the total profit goes to charity and A's share is Rs. 855 then find the total profit?

Or

(b) 72% of the students of a certain class took biology and 44% took Mathematics. If each student took biology or mathematics and 40 took both, find the total number of students in the class.

19. (a) 'A' bought 25 kg of rice at rate of Rs. 6 per kg and 35 kg of rice at the rate of Rs. 7 per kg. He mixed the two and sold the mixture at the rate of Rs. 6.75 per kg. What was his profit or loss in the transaction?

Or

(b) A bought a TV with 20% discount on the labelled price. Had he bought it with 25% discount he would have saved Rs. 500. At what price did he buy the TV?

20. (a) The sum of squares of two numbers is 80. And the square of their difference is 36. Find the product of the numbers.

Or

(b) Of the three numbers, the sum of first two is 45; the sum of the second and the third is 55 and the third and thrice the first is 90. What is the third number?

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B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Third Semester

Mathematics

Skill Based Subject — VECTOR CALCULUS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The directional derivative of $\phi(x,y,z) = x^3 + y^3 + z^3$ at the point (1, –1, 2) is ————

   (a) $3\bar{i} + 4\bar{j} + 3\bar{k}$ (b) $3\bar{i} + 3\bar{j} + 12\bar{k}$

   (c) $3\bar{i} + 3\bar{j} + 3\bar{k}$ (d) $3\bar{i} + 2\bar{j} + 2\bar{k}$

2. The unit vector normal to the surface $\phi = C$ is ————

   (a) $\frac{\nabla\phi}{|\nabla\phi|}$ (b) $\nabla\phi$

   (c) $\nabla^2\phi$ (d) $\frac{|\nabla\phi|}{\nabla\phi}$

3. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, then $\nabla.\bar{r} =$ ————

   (a) $2x$ (b) $3y$

   (c) 3 (d) 4

4. If the vector $(2x,z)\bar{i} + (4x - 11y + 3z)\bar{j} + (3x + mz)\bar{k}$ is solenoidal, then the value of m is ————

   (a) 3 (b) 9

   (c) 2 (d) 11

5. If $\bar{f} = x^2\bar{i} - xy\bar{j}$ and C is the straight line joining the points (0, 0) and (1, 1), then $\int_C \bar{f}.d\bar{r}$ is ————

   (a) 1 (b) 0

   (c) –1 (d) 2

6. If $\overline{F} = z(x\overline{i} + y\overline{j} + z\overline{k})$ and C is the straight line joint (0, 0, 0) and (1, 1, 1), then $\int_C \overline{f}.d\overline{r}$ is ——————

(a) 0 (b) −1

(c) 1 (d) 2

7. If S is any closed surface enclosing a volume V and $\overline{f} = ax\overline{i} + by\overline{j} + cz\overline{k}$, then $\iint_S \overline{f}.\overline{n}\,dS =$ ——————

(a) $(a+b+c)V$ (b) $3V$

(c) $(a+b+c)^3V^3$ (d) 0

8. The value of $\int_0^a\int_0^a\int_0^a x^2ydzdydx$ is ——————

(a) $\frac{a^3}{3}$ (b) $\frac{a^4}{5}$

(c) $\frac{a^5}{4}$ (d) $\frac{a^6}{6}$

9. The value of $\int_C (3x+4y)dx + (2x-3y)dy$, where C is the circle $x^2 + y^2 = 4$ is ——————

(a) 4π (b) -8π

(c) 8π (d) 2π

10. The value of $\iint_C \left[(1+y)z\overline{i} + (1+z)x\overline{j} + (1+x)y\overline{k}\right].d\overline{r}$, where C is a closed curve in the plane $x-2y+z=1$ is ——————

(a) 2 (b) −1

(c) 0 (d) 1

PART B — (5 × 5 = 25 marks)

Answer ALL questions by choosing either (a) or (b).

11. (a) Find the directional derivative of $\phi = x + xy^2 + yz^3$ at (0, 1, 1) in the direction of the vector $2\overline{i} + 2\overline{j} - \overline{k}$.

Or

(b) If $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$ and $|\overline{r}| = r$, prove that $\nabla\left(\frac{1}{r}\right) = \frac{-\overline{r}}{r^3}$.

12. (a) If $\overline{A} = axy\overline{i} + (x^2 + 2yz)\overline{j} + y^2\overline{k}$ is irrotational, find the value of 'a'.

Or

(b) Show that $\nabla^2 r^n = n(n+1)r^{n-2}$ where 'n' is a constant.

13. (a) Evaluate $\int_C \overline{F}.d\overline{r}$ where $\overline{F} = yz\overline{i} + xz\overline{j} - xy\overline{k}$ and C is the straight line having end points $O(0,0,0)$ and $P(2,4,8)$.

Or

(b) If $\overline{f} = 3xy\overline{i} - y^3\overline{j}$, compute $\int_C \overline{F}.d\overline{r}$ along $y = 2x^2$ from (0, 0) to (1, 2).

14. (a) If $\overline{A} = curl\overline{F}$, compute $\iint_S \overline{A}.\hat{n}\, dS$ for any closed surface S.

Or

(b) Evaluate $\iiint_V \nabla.\overline{F}\, dV$ if $\overline{F} = x^2\overline{i} + y^2\overline{j} + z^2\overline{k}$ and if V is the volume of the region enclosed by the cube $0 \le x, y, z \le 1$.

15. (a) Evaluate $\int_C xydx - x^2dy$ by converting it into a double integral. If is given that the boundary of the region bounded by the line $y = x$ and the parabola $x^2 = y$.

Or

(b) Evaluate by Green's theorem $\int_C e^{-x}(\sin ydx + \cos ydy)$ where C is the rectangle with vertices $(0,0), (\pi,0), (\pi,\pi/2), (0,\pi/2)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions by choosing either (a) or (b).

16. (a) If $\nabla\phi = (y + y^2 + z^2)\overline{i} + (x + z + 2xy)\overline{j} + (y + 2zx)\overline{k}$ and if $\phi(1,,11) = 3$, find ϕ.

Or

(b) If $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$ and $\overline{r} = r\hat{r}$ show that (i) $\nabla\,(f(r)\overline{r} = rf'(r) + 3f(r)$ (ii) $\nabla \times (f(r)\overline{r}) = \overline{0}$.

17. (a) Find the value of 'm' if $\overline{F} = (6xy + z^3)\overline{i} + (mx^2 - z)\overline{j} + (3xz^2 - y)\overline{k}$ is irrotational. Find also ϕ such that $\overline{F} = \nabla\phi$.

Or

(b) Show that

(i) $(\overline{V}.\nabla)\overline{r} = \overline{V}$

(ii) $(\overline{V} \times \nabla) \times \overline{r} = -2\overline{V}$.

18. (a) If $\overline{f} = (3x^2 + 6y)\overline{i} - 14yz\overline{j} + 20xz^2\overline{k}$, evaluate $\int_C \overline{F}.d\overline{r}$ where C is (i) a curve whose parametric equations are $x = t, y = t^2, z = t^3$ (ii) straight lines OA, AB, BP where A is $(1, 0, 0)$, B is $(1, 1, 0)$, $O(0, 0, 0)$ and P is $(1,1,1)$.

Or

(b) Evaluate $\iint_S \overline{A}.\overline{n}\, dS$ over the surface S of the region bounded by $x^2 + y^2 = 4$, $z = 0$, $z = 3$ if $\overline{A} = 4x\overline{i} - 2y^2\overline{j} + z^2\overline{k}$.

19. (a) Verify Gauss divergence theorem for $\overline{f} = (x^3 - yz)\overline{i} - 2x^2y\overline{j} + 2\overline{k}$ over the cube bounded by $x = 0, y = 0, z = 0$, $x = a, y = a, z = a$.

Or

(b) Verify Gauss divergence theorem for $\overline{A} = a(x + y)\overline{i} + a(y - x)\overline{j} + z^2\overline{k}$ taken over the region V bounded by the upper hemisphere $x^2 + y^2 + z^2 = a^2$ and the plane $z = 0$.

20. (a) Verify Green's theorem for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region R enclosed by the parabolas $y = x^2$ and $y^2 = x$.

Or

(b) Verify Stoke's theorem for $\overline{F} = y^2\overline{i} + y\overline{j} - xz\overline{k}$ over the upper half of the surface of the sphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$.

· ages)

Reg. No. : ....

ᴗde No. : 30352 E **Sub. Code : SAST 21/ AAST 21**

ƆBCS) DEGREE EXAMINATION, APRIL 2022.

Second/Fourth Semester

Mathematics – Allied

STATISTICS – II

ᶠor those who joined in July 2017 onwards)

Ƭhree hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

hoose the correct answer :

isher's index number is ——— of Laspeyer's nd Paasche's index number.

ı) arithmetic mean (b) geometric mean

ː) harmonic mean (d) none

'he factor reversal test is $I_{pq} \times I_{qp} =$ ———

a) $\frac{\sum p_1 q_0}{\sum p_0 q_1}$ (b) $\frac{\sum p_1 q_0}{\sum p_1 q_1}$

c) $\frac{\sum p_1 q_1}{\sum p_0 q_1}$ (d) $\frac{\sum p_1 q_1}{\sum p_0 q_0}$

Ƭhe standard deviation of the sampling listribution of a statistic is known as ———

(a) normal error (b) standard error

(c) type I error (d) type II error

4. The sample is said to be large if its sample size exceeds ———

(a) 100 (b) 50

(c) 40 (d) 30

5. t-distribution was done by ———

(a) W.S. Gosset (b) Karl Pearson

(c) R.A. Fisher (d) Royden

6. The value of χ^2 range from ———

(a) $-\infty$ to ∞ (b) 0 to ∞

(c) -1 to 1 (d) 0 to 1

7. If k denotes number of rows and h denotes number of columns then the mean square value between the rows in two criteria of classification is ———

(a) $\frac{v_1}{(k-1)}$ (b) $\frac{v_2}{(h-1)}$

(c) $\frac{v_1}{(h-k)}$ (d) $\frac{v_2}{(h-k)}$

8. In three criteria of classification the degrees of freedom between the rows is ———

(a) n (b) $n-1$

(c) $(n-2)$ (d) $(n-1)(n-2)$

9. S.Q.C techniques were developed by ———.

(a) W.A. Shewhart (b) A.L. Bowley

(c) Karl Pearson (d) Edgeworth

10. The Upper Control Limit for R chart is ———.

(a) $D_1\overline{R}$ (b) $D_2\overline{R}$

(c) $D_3\overline{R}$ (d) $D_4\overline{R}$

PART B — (5 × 5 = 25 marks)

nswer ALL questions, choosing either (a) or (b).

(a) Find the value of x in the following data if the ratio between Laspeyre's and Paasche's index number is 28:27.

Commodities	p_0	q_0	p_1	q_1
A	1	10	2	5
B	1	5	x	2

Or

(b) From the following data construct an index number for 1970 taking 1969 as the base by price relatives method using (i) A.M (ii) G.M for averaging the relatives.

Commodities	Price in 1969 Rs.	Price in 1970 Rs.
A	150	170
B	40	60
C	80	90
D	100	120
E	20	25

(a) A sample of 100 tyres is taken from a lot. The mean life of tyres is found to be 39,350 kms, with a standard deviation of 3,260. Could the sample come from a population with mean life of 40,000 kms? Establish 99% confidence limits within which the mean life of tyres is expected to lie.

Or

(b) Intelligence test on two groups of boys and girls gave the following results :

	Mean	S.D	N
Girls	75	15	150
Boys	70	20	250

Is there a significant difference in the mean scores obtained by boys and girls?

13. (a) A random sample of size 16 has 53 as mean. The sum of the squares of the deviations taken from mean is 135. Can this sample be regarded as taken from the population having 56 as mean?

Or

(b) The mean life of a sample of 10 electric light bulbs was found to be 1456 hours with S.d of 423 hours. A second sample of 17 bulbs chosen from a different batch showed a mean life of 1280 hours with S.d of 398 hours. Is there a significant difference between the means of the two samples?

14. (a) The yields of 3 varieties of wheat in 3 blocks are given below. Is the difference between the varieties significant?

Variety \ Block	1	2	3
A	10	9	8
B	7	7	7
C	8	5	4

Or

(b) Write a short note on two criteria of classification.

[P.T.O.]

15. (a) The following table gives the inspection data on completed spark plugs.

(2000 Spark plugs in 20 lots of 100 each)

Lot Number	Number Defectives	Fraction Defectives
1	5	0.50
2	10	0.100
3	12	0.120
4	8	0.080
5	6	0.060
6	5	0.050
7	6	0.060
8	3	0.030
9	3	0.030
10	5	0.050
11	4	0.040
12	7	0.070
13	8	0.080
14	2	0.020
15	3	0.030
16	4	0.040
17	5	0.050
18	8	0.080
19	6	0.060
20	10	0.100

Construct p-chart.

Or

(b) Explain – Acceptance sampling.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Calculate :

(i) Laspeyre's (ii) Paasche's

(iii) Bowley's (iv) Fisher's

(v) Marshall's Edgeworth's

Index numbers for the following data given below

Commodity	Base Year		Curent year	
	Price	Quantity	Price	Quantity
A	2	8	4	6
B	5	10	6	5
C	4	14	5	10
D	2	19	2	15

Or

(b) Construct with a help of data given below. Fisher's index number and show that it satisfies both the factor reversal test and time reversal test

Commodity	A	B	C	D
Base year price in Rupees	5	6	4	3
Base year quantity in Quintals	50	40	120	30
Current year in Rupees	7	8	5	4
Current year quantity in Quintals	60	50	110	35

17. (a) A dice is thrown 9000 times and a throw of 3 or 4 observed 3240 times. Show that the dice cannot be regarded as an unbiased one and find the limits between which the probability of a throw of 3 or 4 lies?

Or

(b) In a random sample of 1,000 persons from town A, 400 are found to be consumers of wheat. In a sample of 800 from town B, 400 are found to be consumers of wheat. Do these data reveal significant difference between town A and town B, so far, as the proportion of wheat consumers is concerned?

18. (a) Two random samples were drawn from two normal populations and their values are

A: 66 67 75 76 82 84 88 90 92

B: 64 66 74 78 82 85 87 92 93 95 97

Test whether the two populations have the same variance at the 5% level of the significance.

Or

(b) In an experiment on pea-breeding Mendel obtained the following frequencies of seeds : 315 round and yellow, 101 wrinkled and yellow, 108 round and green, 32 wrinkled and green. According to his theory of heredity, the numbers should be in proportion 9:3:3:1. Is there any evidence to doubt the theory at 5% level of significance?

19. (a) Analyse the variance in the following Latin square.

B20	C17	D25	A34
A23	D21	C15	B24
D24	A26	B21	C19
C26	B23	A27	D22

Or

(b) There varieties of cows of same age group are treated with four different types of fodders. The yields milk in deciliters are given below. Perform an analysis of variance and check whether is any significant difference between the yields of different varieties of cows due to different types of fodders.

Varieties of cows \ Fodder	f_1	f_2	f_3	f_4
C_1	61	63	66	68
C_2	62	64	67	69
C_3	63	63	68	69

20. (a) Construct $\overline{X}$ and R charts for the following data.

Sample number	Observations		
1	32	37	42
2	28	32	40
3	39	52	28
4	50	42	31
5	42	45	34
6	50	29	21
7	44	52	35
8	22	35	44

Or

(b) Explain control charts and its types.

Reg. No. : ....

Code No. : 20075 E **Sub. Code : SAST 21/ AAST 21**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Second/Fourth Semester

Mathematics — Allied

STATISTICS — II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The ———— mean of Laspeyre's and Paasche's index number is the Bowley's index number.

   (a) arithmetic

   (b) geometric

   (c) harmonic

   (d) none of the above

2. The ———— year is the period against which comprarision is made.

(a) base

(b) current

(c) upcoming

(d) none of the above

3. The standard error of $\bar{x}_1 - \bar{x}_2$ is ————

(a) $\sqrt{\frac{\sigma^2}{2n}}$ (b) $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n^2}}$

(c) $\sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^3}{2n_2}}$ (d) none of the above

4. The standard error of sample variance s^2 is ————

(a) $\frac{\sigma}{\sqrt{n}}$ (b) $\sqrt{\frac{\sigma^2}{2n}}$

(c) $\sigma^2\sqrt{\frac{2}{n}}$ (d) none of the above

5. F test is always ————

(a) two tailed test

(b) right - tailed test

(c) left tailed test

(d) none of the above

6. For the 2×2 contingency table

83	57
45	68

, the value of x^2 =

(a) 10

(b) 9

(c) 9.48

(d) none of the above

7. Analysis of variance (ANOVA) is developed by

(a) Bowley

(b) Kelley

(c) Fisher

(d) None of the above

8. The total degree of freedom for a random sample of N values is

(a) $\frac{N}{2}$ (b) $2N$

(c) $N - 1$ (d) None of the above

9. Control chart is developed by

(a) Shewalt

(b) Taylor

(c) Euler

(d) None of the above

10. The number of horizontal lines in the control chart is ——————

(a) 4 (b) 2

(c) 3 (d) none of the above

PART B — (5 × 5 = 25 marks)

Answer ALL questions by choosing either (a) or (b).

11. (a) Calculate by the weight Aggregate method, the index number from the following data.

Commodity	Base Year (Price per unit)	Current year Price per unit	Weight
Rice	30	40	10
Wheat	20	30	5
Pulses	40	50	6
Oil	35	40	5
Milk	40	50	10

Or

(b) Calculate peice index number for 1945 by (i) Bowley's method and (ii) Fisher's method.

Commodity	1935 Price in Rs.	1935 Quantity	1945 Price in Rs.	1945 Quantity
A	4	50	10	40
B	3	10	9	2
C	2	5	4	2

12. (a) A salesman in a departmental store claims that at most 60% of the shoppers entering the store leave without making a purchase. A random sample of 50 shoppers showed that 35 of them left without making a purchase. Is the sample result consistent with the claim of the salesman? Use a level of significance of 5%.

Or

(b) In a random sample of size 500, the mean is found to be 20. In another independent sample of size 400, the mean is 15. Could the sample have been drawn from the sample population with S.D. 4?

13. (a) The mean height and the S.D. height of 8 randomly chosen boy students are 166.9 cm and 8.29 cm respectively. The corresponding values of 6 randomly chosen girl students are 170.3 cm and 8.50 cm respectively. Based on this data, can we conclude that boy students are, in general, shorter than girl students. (t value for 12 d.f. = 1.782)

Or

(b) A certain injection administered to each of 12 patients resulted in the following change of blood pressure : 5, 2, 8, –1, 3, 0, 6, –2, 1, 5, 0, 4. Can it be concluded that the injection will, in general, result in increase of B.P? (t value = 1.80 for 11 d.f)

14. (a) Explain basic designs of experiment.

Or

(b) Four salesmen were posted in different areas by a company. The number of units of commodity X sold by them are as follows. On the basis of this information can it be concluded that there is a significant difference in the performance of the salesmen. $F_{(3,2)}$ $d.f$ = 3.24

A	20	23	28	29
B	25	32	30	21
C	23	28	35	18
D	15	21	19	25

15. (a) The average number of defectives in 22 sample lots of 2,000 rubber belts was found to be 16%. Obtain the values for central line and control limits for p-chart.

Or

(b) Draw mean chart for the following 10 samples mean of size 5 each 43, 49, 37, 44, 45, 37, 51, 46, 43 and 47. Comment on the state of control of the process. (n = 5, A_2 = 0.58)

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Construct (i) Chain Base Index (ii) Fixed Base Index taking 1993 as origin :

Year	Price (in Rs.) per quintal
1963	50
1964	60
1965	62
1966	65
1967	70
1968	78
1969	82
1970	84
1971	88
1972	90

Or

(b) The table below gives the princes of base year and current year of 5 commodities with their quantities. Use it to verify whether Fishere's index satisfies factor reversal test and time reversal test.

Commodity	Price year		Current year	
	Unit Price (Rs.)	Quantity	Unit Price (Rs.)	Quantity
A	5	50	5	70
B	5	75	10	80
C	10	80	12	100
D	5	20	8	100
E	10	50	5	60

17. (a) On the basis of their total scores, 200 candidates of a civil service examination are divided into two groups, the upper 30 percent and the remaining 70 percent. Consider the first question of this examination..Among the first group, 40 had the correct answer, whereas among the second group. 80 had the correct answer. On the basis of these results, can we conclude that the first question is no good at discriminating ability of the type being examined here.

Or

(b) Random samples drawn from two countries gave the following data relating to the heights of adult males.

	Country A	Country B
Mean height in inches	67.42	67.25
Standard deviation	2.58	2.50
Number in samples	1000	1200

(i) Is the difference between the means significant?

(ii) Is the difference between the standard deviations significant.

18. (a) Fit a poisson distribution to the following data and test the goodness of fit. Also given χ^2 for 2 d.f at 5% level of significance is 5.99.

x	0	1	2	3	4	5	6
f	275	72	30	7	5	2	1

Or

(b) A random sample of 10 boys had the following IQ's : 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I.Q of 100 . Find a reasonable range in which most of the mean IQ values of samples of 10 boys lie.

19. (a) Five types of treatments are given. The number average and standard deviation for each treatment are given in the following table. Test whether the treatments are homogeneous.

	A	B	C	D	E
Treatment No :	10	6	8	11	5
Mean :	10.9	13.5	11.5	11.2	15.4
Standard deviation :	12.72	5.96	3.24	5.65	3.64

Or

(b) Carry out analysis of variance for data of 7 varities, 5 observations being taken on each variety.

Variety No :	1	2	3	4	5	6	7
Observation No :							
1	13	15	14	14	17	15	16
2	11	11	10	10	15	9	12
3	10	13	12	15	14	13	13
4	16	18	13	17	19	14	15
5	12	12	11	10	12	10	12

20. (a) The following table gives the number of defective items found in 20 successive samples of 100 items each.

2	6	2	4	4	15	0	4	10	18
2	4	6	4	8	0	2	2	4	0

Comment whether the process is under control suggest suitable control limits for the future.

Or

(b) Ten pieces of cloth contained the following number of defects : 3, 0, 2, 8, 4, 2, 1, 3, 7, 1. Prepare a C-chart and state whether the production process is in a state of control.

(6 pages) Reg. No. : ....

Code No. : 20083 E **Sub. Code : SEMA 5 B/ AEMA 52**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fifth Semester

Mathematics

Major Elective — DISCRETE MATHEMATICS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL the questions.

Choose the correct answer

1. The statement Q is called the ———— in $P \to Q$
   (a) antecedent (b) consequent
   (c) tautologies (d) None

2. If P and Q are two statements, then the statement $P \to Q$ is called ————
   (a) Conditional statement
   (b) Biconditional statement
   (c) Simple statement
   (d) None

3. Every axiom is a ————
   (a) lemma (b) statement
   (c) theorem (d) formula

4. A sum of the variables and their negations is called on ————.
   (a) elementary sum (b) elementary product
   (c) normal sum (d) none

5. A group $<G,*>$ in which the operation $*$ is commutative is called an ———— group.
   (a) semi (b) subgroup
   (c) abelian (d) none

6. Semigroup $<M,0>$ with an identity element with respect to the operation 0 is called a ————.
   (a) monoid (b) group
   (c) abelian (d) permutation

7. Every ———— is a distributive lattice.
   (a) chain (b) group
   (c) bounds (d) homomorphism

8. A ———— algebra is a complemented, distributive lattice.
   (a) Boolean (b) Partial
   (c) Ordinary (d) None

9. What are the numbers using for represent octal number?
   (a) 0-9 (b) 0-1
   (c) 0-7 (d) none

10. Subtract 01110 from 10101?
    (a) 11001 (b) 100100
    (c) 00111 (d) 11001

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Construct the truth table for Biconditional.

    Or

    (b) Construct the truth table for $(P \vee Q) \vee \urcorner P$

12. (a) Symbolize the statement: "All men are giants".

    Or

    (b) Prove that $(fx)(P(x) \wedge Q(x)) \Rightarrow (fx)P(x) \wedge (fx)Q(x)$.

13. (a) Show that the kernel of a homomorphism g from a group $<G,*>$ to $<H,\Delta>$ is a subgroup of $<G,*>$.

    Or

    (b) Prove that the parity – check matrix H generates a code word of weight q if there exists a set of q columns of H such that their k-tuple sum is zero.

14. (a) Let $<L,*,\oplus>$ be a distributive lattices. Then prove that for any $a,b,c,\in L,(a*b=a*c)$ $\wedge(a \oplus b = a \oplus c) \Rightarrow b = c$

    Or

    (b) Prove the Boolean identify $(A+B)(A+C) = A+BC$.

15. (a) Convert $(101010101)_2$ to hexadecimal.

    Or

    (b) Divide 100001 by 110?

[P.T.O.]

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Construct the truth table for $(Q \wedge (P \rightarrow Q)) \rightarrow P$.

Or

(b) Does the formula $(\neg Q \wedge P) \wedge Q$ is tautology or not.

17. (a) Obtain the principal disjunctive norm forms of

(i) $\neg P \vee Q$,

(ii) $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

Or

(b) Does P follows from $(P \vee Q)$.

18. (a) Write a definition of Group and subgroup.

Or

(b) Prove that the minimum weight of the nonzero code words in a group code is equal to minimum distance.

19. (a) If any Boolean algebra, show that $a = b \Leftrightarrow ab' + a'b = 0$.

Or

(b) When $< B, *, \oplus >$ becomes a lattice.

20. (a) Convert the following to octal numbers (i) 110101110_2 (ii) 111101.01101_2

Or

(b) Multiply :

(i) 1.01×10.1

(ii) 100101×1001

(8 pages)

Reg. No. : ....

Code No. : 20085 E **Sub. Code : SEMA 5 D/ AEMA 54**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fifth Semester

Mathematics

Major Elective — OPERATIONS RESEARCH – I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Linear programming problem involving only two decision variables can be solved by
   (a) Graphical method
   (b) Simplex method
   (c) Both (a) and (b)
   (d) None of the above

2. Simplex method was introduced by
   (a) G.B Dantzig (b) Konig
   (c) Miller (d) Taha

3. If the primal problem has an unbounded solution then the dual problem has ————
   (a) feasible solution
   (b) basic solution
   (c) no feasible solution
   (d) optimal solution

4. If the dual problem has n variables, then the primal problem has ———— constraints.
   (a) n (b) $n+1$
   (c) $n-1$ (d) none

5. All the basis for a transportation problem are
   (a) triangular (b) non-triangular
   (c) equal (d) unequal

6. Which of the following method used to find the optimum solution of transportation problem?
   (a) VAM method
   (b) North-West corner rule
   (c) MODI method
   (d) Matrixminima method

7. In a balanced assignment problem the cost matrix is
   (a) Symmetric matrix
   (b) Square matrix
   (c) Unsymmetric matrix
   (d) Not a square matrix

8. If an assignment problem having 4 workers and 3 jobs, then the total number of possible assignment is
   (a) 4 (b) 3
   (c) 7 (d) 12

9. If indicates the time required by a job on each machine
   (a) Elapsed time (b) Processing time
   (c) Idle time (d) None

10. Number of sequences require to evaluate sequencing problem with 6 jobs, 5 machines
    (a) $(6!)^5$ (b) $(5!)^6$
    (c) $5!\times 6$ (d) 5×6

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Solve the following LPP using graphical method

Maximize $z = x_1 + x_2$

S.T

$x_1 + x_2 \le 1$

$-3x_1 + x_2 \ge 3$

$x_1, x_2 \ge 0.$

Or

(b) Solve the following LPP, using graphical method

Minimize $z = x_1 + x_2$

S.t

$5x_1 + 3x_2 \le 15$

$x_1 + x_2 \ge 6$

$x_1, x_2 \ge 0.$

12. (a) Write the dual of the following LPP.

Minimize $z = 4x_1 + 6x_2 + 18x3$

S.t

$x_1 + 3x_2 \geq 3$

$x_2 + 2x_3 \geq 5$

$x_1, x_2, x_3 \geq 0.$

Or

(b) Prove that the dual of the dual is primal.

13. (a) Explain the North - West corner rule.

Or

(b) Obtain an initial basic feasible solution to the following transportation problem.

	D	E	F	G	
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
	200	225	275	250	

14. (a) Write the mathematical formulation of the assignment problem.

Or

(b) State and prove reduction theorem.

15. (a) Explain the basic terms used in sequencing problem.

Or

(b) Find the optimum sequence of the following data.

Jobs :	J_1	J_2	J_3	J_4	J_5	J_6
Machine A :	1	3	8	5	6	3
Machine B :	5	6	3	2	2	10

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Use penalty method to solve the LPP.

Maximize $z = 6x_1 + 4x_2$

S.t

$2x_1 + 3x_2 \leq 30$

$3x_1 + 2x_2 \leq 24$

$x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0.$

Or

(b) Explain the simplex algorithm.

17. (a) Use duality to solve

Maximize $z = 2x_1 + x_2$

S.t.

$x_1 + x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1 - 2x_2 \leq 1$

$x_1, x_2 \geq 0.$

Or

(b) Explain the dual simplex method.

18. (a) Write the transportation algorithm in detail.

Or

(b) Solve the following transportation problem.

	D_1	D_2	D_3	D_4	
S_1	3	7	6	4	5
S_2	2	4	3	2	2
S_3	4	3	8	5	3
	3	3	2	2	

19. (a) Write the Hungarian algorithm.

Or

(b) Solve the following assignment problem.

	1	2	3
I	9	26	15
II	13	27	6
III	35	20	15
IV	18	30	20

20. (a) Solve the following sequencing problem.

Jobs :	A	B	C	D	E	F	G	H	I
M_1:	2	5	4	9	6	3	7	5	4
M_2:	6	8	7	4	3	9	3	8	11

Or

(b) Explain the processing 2 jobs through K machines.

———————

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ode No. : 30366 E **Sub. Code : SEMA 6 B**

(CBCS) DEGREE EXAMINATION, APRIL 2022.

Sixth Semester

Mathematics

Major Elective — FUZZY MATHEMATICS

(For those who joined in July 2017 onwards)

ne : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

Which of the following symbol is used for the existential quantifier?

(a) $\forall$ (b) $\exists$

(c) iff (d) α

$\alpha_A = \{x / A(x) \geq \alpha\}$ is

(a) strong α-cut (b) α-cut

(c) crisp set (d) fuzzy set

3. Let $A, B, \in \mathcal{F}(x)$ and $\alpha, \beta \in [0, 1]$, then $\alpha \leq \beta \Rightarrow$

(a) $\alpha_A = \beta_A$ (b) $\alpha_A \supseteq \beta_A$

(c) $\alpha_A \subseteq \beta_A$ (d) None

4. Third decomposition theorem states

(a) $A = \bigcup_{\alpha \in A(A)} \alpha A$ (b) $A = \bigcup_{\alpha \in [0, 1]} \alpha A$

(c) $A = \bigcup_{\alpha \in [0, 1]} \alpha + A$ (d) None

5. The value of $(A \cap B)x =$

(a) $\max[A(x), B(x)]$

(b) $\min[A(x), B(x)]$

(c) $\max[\overline{A}(x), \overline{B}(x)]$

(d) $\min[\overline{A}(x), \overline{B}(x)]$

6. The value of $c(A(x))$

(a) $A\,c(x)$

(b) $A\ c((x))$

(c) $c\,A(x)$

(d) None

$A \subseteq E$ and $B \subseteq F$ then

ı) $A + B \subseteq E + F$

ɔ) $A + E \subseteq B + F$

:) $E + F \subseteq A + B$

l) $A + F \subseteq B + E$

'he value of MIN$[A$, MAX$(A, B)]$

ı) MAX (A, B)

ɔ) MIN(A, B)

:) A

l) None

ı fuzzy model group decision was proposed by

a) Bellman (b) Blin

:) Robert (d) Dantzig

ı fuzzy decision making was introduced at

a) 1970 (b) 1977

c) 1980 (d) 1972

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Write any ten fundamental properties of crisp set operations.

Or

(b) Define the following with example.

(i) crisp set

(ii) fuzzy set.

12. (a) Let $A, B \in \mathcal{F}(x)$ and $\alpha, \beta \in [0, 1]$ then prove that $\alpha(\overline{A}) = (1 - \alpha) + \overline{A}$.

Or

(b) Let $f : x \to y$ be an arbitrary crisp function. Then for any $A \in \mathcal{F}(x)$ and all $\alpha \in [0, 1]$, prove that $\alpha + [f(A)] = f(\alpha + A)$.

13. (a) If a complement c has an equilibrium e_c, then prove that $d_{e_c} = e_{\hat{c}}$.

Or

(b) Prove that the standard fuzzy intersection is the only idempotent t-norm.

14. (a) Write a short note on fuzzy numbers.

Or

(b) Explain the fuzzy equations.

15. (a) Explain the individual decision making.

Or

(b) Explain the fuzzy linear programming.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Explain the crisp sets in detail.

Or

(b) Prove that a fuzzy set A on R is convex iff $A(\lambda x_1 + (1-\lambda)x_2) \geq \min[A(x_1), A(x_2)]$ for all $x_1, x_2 \in R$ and all $\lambda \in [0,1]$.

17. (a) Let $A_i \in \mathcal{F}(x)$ for all $i \in I$. Then prove that $\bigcup_{i\in I} \alpha + A_i = \alpha + \left(\bigcup_{i\in I} A_i\right)$ and $\bigcap_{i\in I} \alpha + A_i \subseteq \alpha + \left(\bigcap_{i=I} A_i\right)$.

Or

(b) State and prove first decomposition theorem.

18. (a) State and prove second characterization theorem of fuzzy complements.

Or

(b) For all $a, b \in [0,1]$ then prove that $i_{\min}(a,b) \leq i(a,b) \leq \min(a,b)$.

19. (a) For any $A, B, C \in R$ prove that $\mathrm{MIN}[\mathrm{MIN}(A,B), C] = \mathrm{MIN}[A, \mathrm{MIN}(B,C)]$.

Or

(b) Let $A \in \{+, -, \cdot, /\}$ and let A, B denote continuous fuzzy numbers, then prove that the fuzzy set $A * B$ defined by $(A*B)(z) = \sup_{z=x+y} \min[A(x), B(y)]$ is a continuous fuzzy number.

20. (a) Explain the multi person decision making.

Or

(b) Solve the fuzzy linear programming problem.

$$\max = z = 5x_1 + 4x_2$$

$$s.t. (4,2,1)x_1 + (5,3,1)x_2 \leq (24,5,8)$$

$$(4,1,2)x_1 + (1,.5,1)x_2 \leq (12,6,3)$$

$$x_1, x_2 \geq 0.$$

) A company operating 50 weeks in a year is concerned about its stocks of copper cable. This costs Rs. 240 a metre and there is a demand for 8,000 meteres a week. Each replenishment costs Rs. 1,050 for administration and Rs. 1,650 for delivery, while holding costs are estimated at 25 percent of value held a year. Assume no shortages are allowed, what is the optimal inventory policy for the company? How would this analysis differ if the company wanted to maximize profit rather than minimize cost? What is the gross profit if the company sell cable for Rs. 360 a metre.

Or

The demand for an item in a company is 18,000 units per year, and the company can produce the items at a rate of 3,000 per month. The costs of one set-up is Rs. 500 and holding cost of 1 unit per month is 15 paise. The storage cost of one unit is Rs. 200 per month. Determine (i) optimum production batch quantity and the number of strategies, (ii) optimum cycle time and the production time, (iii) maximum inventory level in the cycle and (iv) total associated cost per year of the cost of the time is Rs. 20 per unit.

———————

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B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2022

Sixth Semester

Mathematics

Major Elective — OPERATIONS RESEARCH - II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. A game is said be fair, if ————

   (a) upper value is more than lower value of the game

   (b) upper and lower values are not equal

   (c) upper and lower values are same and zero

   (d) upper value is less than lower value of the game

2. The size of the pay-off matrix of a game can be reduced by using the principle of ————

(a) dominance (b) rotation reduction

(c) game inversion (d) game transpose

3. The problem of replacement is not concerned about the ————

(a) items are deteriorate graphically

(b) determination of optimum replacement interval

(c) items that fails suddenly

(d) maintenance of an item to work out profitability

4. Mortality problems ————

(a) are special type of problems, where failure is treated as birth and the replacement of an item on the failure is treated as death

(b) uses mortality tables to derive the probability distribution of the life span of an equipment/item

(c) are like replacement policies for items whose value does not deteriorate gradually

(d) none of the above

(i) Find the critical path and the expected time of the project.

(ii) Find the total and free-float for each activity.

Or

(b) A project is composed of eight activities, the time estimates for which are given below.

Activity		Time required (days)		
Event	Name	t_o	t_m	t_p
1-2	A	6	6	24
1-3	B	6	12	18
1-4	C	12	12	30
2-5	D	6	6	6
3-5	E	12	30	48
4-6	F	12	30	42
5-6	G	18	30	54

(i) Find the expected duration and variance of each activity.

(ii) What is the expected project length?

(iii) Calculate the variance and standard deviation of the project length.

If the tubes are group replaced, the cost of replacement is Rs. 15 per tube. Group replacement can be done at fixed intervals at fixed intervals in the night shift when the computer is not normally used. Replacement of individual tubes which fail in services costs Rs. 60 per tube. How frequently should the tubes be replaced?

a) Explain the solution procedure of the queueing model (M/M/1) : (∞/FIFO). Also obtain its characteristics.

Or

b) Explain the queueing model (M/M/1) : (N/FIFO) and find its characteristics.

a) The following are the details of estimated times of activities of a certain project.

Activity	Immediate predecessors	Normal time (days)
A	–	16
B	–	20
C	A	8
D	A	10
E	B,C	6
F	D,E	12

5. Queue can form only when ————

(a) arrivals exceed service capacity

(b) arrivals equals service capacity

(c) service facility is capable to serve all the arrivals at a time

(d) there are more than one service facilities

6. Priority queue discipline may be classified as ————

(a) finite or infinite

(b) limited or unlimited

(c) pre-emptive or non-pre-emptive

(d) all of the above

7. Network problems have advantage in terms of project ————

(a) scheduling (b) planning

(c) controlling (d) all of the above

8. In critical path analysis, CPM is ————

(a) event oriented

(b) probabilistic in nature

(c) deterministic in nature

(d) dynamic in nature

9. Which of the following is not an assumption underlying the fundamental problem of EOQ?

(a) demand is known and uniform

(b) lead time is not zero

(c) holding cost per unit time period is constant

(d) stock-outs are not permitted

10. If the procurement cost used in the formula to compute EOQ is half of the actual procurement cost, the EOQ so obtained will be ———

(a) half of EOQ (b) 0.707 time EOQ

(c) one third of EOQ (d) one forth of EOQ

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Solve the game whose payoff matrix is given by

Player B

Player A		B_1	B_2	B_3
	A_1	1	3	1
	A_2	0	−4	−3
	A_3	1	5	−1

Or

17. (a) A manufacturer is offered two machines A and B. A is priced at Rs. 5,000 and running costs are estimated at Rs. 800 for each of the first years, increasing by Rs. 200 per year in the sixth and subsequent years. Machine B which has the same capacity as A costs Rs. 2,500 but will have running costs of Rs. 1,200 per year for six years, increasing by Rs. 200 per year thereafter. If money is worth 10% per year, which machine should be purchased? (Assume that the machine will eventually be sold for scrap at a negligible price)

Or

(b) A computer has a large number of electronic tubes. They are subject to mortality as given below.

Period	Age of failure (hours)	Probability of failure
1	0-200	0.10
2	201-400	0.26
3	401-600	0.35
4	601-800	0.22
5	801-1000	0.07

) The demand for a certain items is 16 units per period. Unsatisfied demand causes a shortage cost of Re. 0.75 per unit per short period. The cost of initializing purchasing action is Rs. 15.00 per purchase and the holding cost is 15% of average inventory valuation per period. Item cost is Rs. 8.00 per unit. (Assume that shortages are being back ordered at the above mentioned cost). Find the minimum cost purchase quantity.

PART C — (5 × 8 = 40 marks)

swer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

ı) Use graphical method in solving the following game :

Player A

$$\text{Player B}\begin{pmatrix} 2 & 2 & 3 & -2 \\ 4 & 3 & 2 & 6 \end{pmatrix}$$

Or

) Solve the following game by using dominance property.

Player A

$$\text{Player B}\; \begin{matrix} \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \end{matrix} \begin{pmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{pmatrix}$$

(b) For the game with the following payoff matrix, determine the optimum strategies and the value of the game :

P_2

$$P_1 \begin{pmatrix} 5 & 1 \\ 3 & 4 \end{pmatrix}$$

12. (a) A firm is considering replacement of a machine, whose cost price is Rs. 12,200 and the scrap value, Rs. 200.

The running (maintenance and operating) cost in rupees are found from experience to be as follows :

Years :	1	2	3	4
Running cost :	200	500	800	1,200
Years :	5	6	7	8
Running cost :	1,800	2,500	3,200	4,000

When should the machine be replaced?

Or

(b) The cost of new machine is Rs. 15,000. The maintenance cost of n^{th} year is given by

C_n = 500(n-1); n = 1, 2,.. suppose that the discount rate per year is 0.5. After how many years if will be economical to replace the machine by a new one?

13. (a) A T.V. repairman finds that the time spent on his jobs has an exponential distribution with 30 minutes. If he repairs sets in the order in which they came in and if the arrival of sets is approximately.

Poisson with an average rate of 10 per 8 hour day. What is repairman's expected idle time each day? How much jobs are ahead of the average set just brought in?

Or

(b) At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an average of 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady-state probabilities for the various number of trains in the system. Also find the average waiting time of a new train coming into the yard.

14. (a) Given the following information :

Activity :	0-1	1-2	1-3	2-4	2-5
Duration (in days) :	2	8	10	6	3
Activity :	3-4	3-6	4-7	5-7	6-7
Duration (in days) :	3	7	5	2	8

(i) Draw the arrow diagram.

(ii) Identify critical path and find the project duration.

Or

(b) Consider the data of the project, find its critical path and project duration.

Activity	A	B	C	D	E	F	G	H	I
Predecessor	–	–	A	B	C,D	B	E	E	F,G
Duration (days)	4	7	2	9	6	5	2	10	4

15. (a) A company plans to consume 760 pieces of a particular component. Past records indicate that purchasing department had used Rs. 12,000 for placing 15,000 orders. The average inventory was valued at Rs. 45,000 and the total storage cost was Rs. 7,650 which included wages, rent, taxes, insurance, etc., related to store department. The company borrows capital at the rate of 10% a year. If the price of a component is Rs. 12 and the order size is of 10 component, determine; purchase cost, purchase expenses, storage expenses, capital cost and total cost per year.

Or

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Code No. : 20091 E **Sub. Code : SEMA 6 D**

B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Sixth Semester

Mathematics

Major Elective — OPERATIONS RESEARCH – II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The pay-off value for which each player in a game always selects the same strategy is called the ————

   (a) equilibrium point

   (b) saddle point

   (c) both (a) and (b)

   (d) maximum point

2. A mixed strategy game can be solved by ————

(a) matrix method

(b) algebraic method

(c) graphical method

(d) all of the above

3. For the items that deteriorate gradually

(a) Operating and maintenance costs steadily increase with passage of time, where as depreciation per year decrease with time

(b) Optimum replacement interval is the minimum time elapsing between the successive replacements

(c) The annual maintenance cost and annual depreciation tend to decrease

(d) All of the above

4. Staff replacement policy is ————

(a) arises due to resignation, retirement or death of a staff member from time to time

(b) is like replacement policy for items whose values deteriorate gradually

(c) can be easily formulated because people retire at known times

(d) does not yield the optimum replacement interval

5. When there are more than one servers, customer behaviour in which he moves from one queue to another is known as ————

(a) balking (b) jockeying

(c) reneging (d) alternating

6. Which of the following is not a key operating characteristic for a queuing system

(a) average time a customer spent waiting in the system and queue

(b) utilization factor

(c) percent idle time

(d) none of the above

7. The main objective of network analysis is to ————

(a) minimize total project cost

(b) minimize total project duration

(c) minimize production delays, interruption and conflicts

(d) all of the above

8. The slack for an activity in network, is equal to

(a) LS - ES (b) LF - LS

(c) EF - ES (d) EF - LS

9. Economic Order Quantity (EOQ) results in ——

(a) equilisation of carrying cost and procurement cost

(b) minimization of set up cost

(c) favourable procurement price

(d) reduced chances of stock outs

10. If small orders are placed frequently, then total inventory cost ——

(a) decreased

(b) increased

(c) either decreased or increased

(d) no change

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Consider the game with the following pay off matrix

$$A \overset{B}{\begin{pmatrix} 0 & 2 \\ -1 & 4 \end{pmatrix}}$$

Show that the game is strictly determinable. Also find the value of the game.

Or

(b) Consider a 'modified' form of 'matching biased coins' game problem. The matching player is paid Rs. 8.00 if the two coins turn both heads and Rs. 1.00 if the coins turn both tails. the non-matching player paid Rs. 3.00 when the two coins do not match. Given the choice of being the matching or non-matching player, which one would you choose and what would be your strategy?

12. (a) The data collected in running a machine, the cost of which is Rs. 60,000, are given below :

Year	1	2	3	4	5
Resale value (Rs.) :	42,000	30,000	20,400	14,400	9,000
Cost of spares (Rs.) :	4,000	4,270	4,880	5,700	6,800
Cost of labour (Rs) :	14,000	16,000	18,000	21,000	25,000

Determine the optimum period for replacement of the machine.

Or

(b) A pipeline is due for repairs. It will cost Rs. 10,000 and last for 3 years. Alternatively, a new pipeline can be laid at a cost of Rs. 30,000 and lasts for 10 years. Assuming cost of capital to be 10% and ignoring salvage value, which alternative should be chosen?

13. (a) In a railway marshaling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. Calculate the following : (i) the mean queen size (line length), (ii) the probability that the queue size exceed 10.

Or

(b) A super market has two girls serving at the counters. The customers arrive in a Poisson fashion at the rate of 12 per hour. The service time for each customer is exponential with mean 6 minutes. Find (i) the probability that an arriving customer has to wait for service (ii) the average number of customers in the system.

14. (a) Draw a network diagram for the following relation ships :

Activity :	A	B	C	D	E	F	G
Immediate predecessor :	–	–	–	A	B,C	A	C
Activity :	H	I	J	K	L	M	N
Immediate predecessor :	F,E,D	D	G	G	J,H	K	I,L

Or

(b) Given the following information :

Activity :	0-1	1-2	1-3	2-4	2-5
Duration (in days) :	2	8	10	6	3
Activity :	3-4	3-6	4-7	5-7	6-7
Duration (in days) :	3	7	5	2	8

(i) Draw the arrow diagram

(ii) Identify critical path and find the total project duration.

15. (a) A manufacturing company purchases 9,000 parts of a machine for its annual requirements, ordering one month usage at a time. Each part costs Rs. 20. The ordering cost per order is Rs. 15 and the carrying charges are 15% of the average inventory per year. You have been assigned to suggest a more economical purchasing policy for the company. What advice would you offer and how much would it save the company per year?

Or

(b) A contractor under takes to supply diesel engines to a truck manufacturer at the rate of 25 per day. There is a clause in the contract penalizing him Rs. 10 per engine per day late for missing the scheduled delivery date. He finds that the cost of holding a complete engine in stock is Rs. 16 per month. His production process is such that each month he starts a batch of engines through the shops, and all these engines are available for delivery any time after the end of the month. What should his inventory level be at the beginning of each month?

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Obtain the optimal strategies for both - persons and the value of the game for zero - sum - two - person game whose pay-off matrix is as follows :

$$\begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix}$$

Or

(b) Two firms are competing for business under the condition so that one firm's gain is another firm's loss. Firm A's payoff matrix is given below :

Firm A \ Firm B	No ad	Med.ad.	Heavy ad.
No advertising	10	15	–2
Medium advertising	13	12	15
Heavy advertising	16	14	10

Suggest optimum strategies for the two firms and the net outcome thereof.

17. (a) (i) Machine A costs Rs. 9,000. Annual operating costs are Rs. 200 for the first year, and then increase by Rs. 2,000 every year. Determine the best age at which to replace the machine. If the optimum replacement policy is followed, what will be the average yearly cost of owning and operating the machine?

(ii) Machine B cost Rs. 10,000. Annual operating costs are Rs. 400 for the first year, and then increase by Rs. 800 every year. You now have a machine of type A which is one-year old, should you replace its with B; if so when?

Or

(b) A computer has a large number of electric tubes. They are subject to mortality as given below :

Period	Age of failure (hours)	Probability of failure
1	0-200	0.10
2	201-400	0.26
3	401-600	0.35
4	601-800	0.22
5	801-1000	0.07

If tubes are group replaced, the cost of replacement is Rs. 15 per tube. Group replacement can be done at fixed intervals in the night shift when the computer is not normally used. Replacement of individual tubes which fail in service costs Rs. 60 per tube. How frequently should the tubes be replaced.

18. (a) The arrival rate of customers at a public telephone booth follows. Poisson distribution with an average time of 10 minutes between one customer and the next. The duration of phone call is assumed to follow exponential distribution, with mean time of 3 minutes.

(i) What is the probability that a person arriving at the booth will have to wait?

(ii) What is the average length of the non-empty queues that form from time to time?

(iii) The Mahanagar telephone Nigam Ltd., will install a second booth when it is convinced that the customers would expect waiting for atleast 3 minutes for their turn to make a call. By how much time should the flow of customers increase in order to justify a second booth?

(iv) Estimate the fraction of a day that the phone will be in use.

Or

(b) A bank has tow tellers working on savings accounts. The first seller handles withdrawals only. The second teller handles deposits only. If has been found that the service time distribution for booth deposits withdrawals is exponential with mean service time 3 minutes per customer. Depositors are found to arrive in Poisson fashion throughout the day with mean arrival rate of 16 per hour. Withdrawers also arrive in Poisson fashion with mean arrival rate of 14 per hour. What would be the effect on the average waiting time for depositors and withdrawers if each seller could handle both withdrawals and deposits? What could be the effect if this could be accomplished by increasing the mean service time to 3.5 minutes?

19. (a) A project consists of a series of tasks labeled A, B, . . .,H,I with the following relationship (W < X, Y means X and Y cannot start until both X and Y are completed) . With this notation construct the network diagram having the following constraints : A < D, E; B, D < F; C < G; B, G <H; F, G < I. Find also the minimum time of completion of the project, when the time (in days) of completion of each task is as follows :

Task :	A	B	C	D	E	F	G	H	I
Time :	23	8	20	16	24	18	19	4	10

Or

(b) A small project is composed of seven activities whose time estimates are listed in the table as follows :

Activity		Estimated duration		
i	j	Optimistic	Most likely	Pessimistic
1	2	1	1	7
1	3	1	4	7
1	4	2	2	8
2	5	1	1	1
3	5	2	5	14
4	6	2	5	8
5	6	3	6	15

(i) Draw the project network

(ii) Find the expected duration and variance of each activity. What is the expected project length?

(iii) Calculate the variance and standard deviation of project length.

What is the probability that the project will be completed :

(1) atleast 4 weeks earlier than expected?

(2) no more than 4 weeks later than expected?

20. (a) A company operating 50 weeks in a year is concerned about its stocks of copper cable. This costs Rs. 240 a metre and there is a demand for 8,000 metres a week. Each replenishment costs Rs. 1,050 for administration and Rs. 1,650 for delivery, which holding costs are estimated at 25 percent of value held a year. Assuming no shortages are allowed, What is the optimal inventory policy for the company? How would this analysis differ if the company wanted to maximize profit rather than minimize cost? What is the gross profit if the company sell cable for Rs. 360 a metre.

Or

(b) A dealer supplies you the following information with regard to a product dealt in by him :

Annual demand - 10,000 units; ordering cost - Rs. 10 per order; price - Rs. 20 per unit.

Inventory carrying cost - 20% of the value of inventory per year.

The dealer is considering the possibility of allowing some balk - order (stock - out) to occur. He has estimated that the annual cost of back - ordering will be 25% of the value of inventory.

(i) What should be the optimum number of units of the product he should buy in one lot?

(ii) What quantity of the product should be allowed to be back-ordered, if any?

(iii) What would be the maximum quantity of inventory at any time of the year?

(iv) Would you recommend to allow back-ordering?

If so, what would be the annual cost saving by adopting the policy of back - ordering?

pages) **Reg. No. :** ....

Code No. : 20065 E **Sub. Code : SMMA 53/ AMMA 53**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fifth Semester

Mathematics – Core

STATICS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

If the angle between two equal forces P and P is α, then their resultant is ————

(a) $2P$

(b) $2P\cos\alpha$

(c) $2P\cos\frac{\alpha}{2}$

(d) 0

2. If the resultant of two forces P and Q is at right angle to P, the angle between the forces is ————

(a) $\cos^{-1}(PQ)$ (b) $\cos^{-1}\left(\frac{P}{Q}\right)$

(c) $\cos^{-1}(-PQ)$ (d) $\cos^{-1}\left(-\frac{P}{Q}\right)$

3. Two paraller forces acting in the same direction are called ———— forces.

(a) Like (b) Unlike

(c) Direct (d) Opposite

4. The magnitude of the resultant of two unlike parallel forces is their ————

(a) difference (b) Sum

(c) multiplication (d) ratio

5. If three coplanar forces acting on a rigid body keep it in equilibrium, then they must be ————

(a) concurrent (b) parallel

(c) either (a) or (b) (d) zero

6. The coefficient of friction $\mu =$

(a) $\frac{F}{R}$ (b) FR

(c) $\tan^{-1}\left(\frac{F}{R}\right)$ (d) $\tan^{-1}(FR)$

The relation between the coefficient of friction and the angle of friction is ————

(a) tan $\mu = \lambda$ (b) tan $\lambda = \mu$

(c) tan $(\lambda\mu) = 1$ (d) $\tan\lambda = \frac{1}{\mu}$

The angle of repose of a rough inclined plane = ————

(a) 0 (b) μ

(c) λ (d) $\tan^{-1}\lambda$

The intrinsic equation of the catenary is ————

(a) $s = c\tan\psi$

(b) $s = \tan\psi$

(c) $s = c\tan\left(\frac{x}{c}\right)$

(d) $s = \tan h\left(\frac{x}{c}\right)$

If the weight per unit length of the chain is constant, then the caternary is called the ———— catenary.

(a) constant (b) same

(c) common (d) unique

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove the triangle law of forces.

Or

(b) Two forces act on a particle. If the sum and difference of the forces are at right angles to each other, show that the forces are of equal magnitude.

12. (a) Derive the condition of equilibrium of three coplanar paraller forces.

Or

(b) Three like parallel forces, acting at the vertices of a triangle, have magnitudes proportional to the opposite sides. Show that their resultant passes through the incentre of the triangle.

13. (a) State the procedure to be followed in solving any statical problem.

Or

(b) A heavy uniform rod of length 2a rests partly within and partly without a smooth hemispherical bowl of radius r, fixed with its rim horizontal. If α is the inclination of the rod to the horizon, show that $2r\cos 2\alpha = a\cos\alpha$.

[P.T.O.]

14. (a) State the laws of friction.

Or

(b) Write a short note on:

(i) Angle of friction

(ii) Cone of friction

15. (a) Derive the cartesian equation of a catenary.

Or

(b) State and prove any one geometrical property of a common catenary.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Lami's theorem.

Or

(b) If 0 is the circumcentre of the triangle ABC and the forces P,Q,R acting along the lines OA, OB, OC respectively are in equilibrium then prove that

$$P : Q : R = a^2\left(b^2+c^2-a^2\right) : b^2\left(a^2+c^2-b^2\right) : c^2\left(a^2+b^2-c^2\right)$$

17. (a) Force P,Q, R act along the sides BC, AC, BA respectively of an equilateral triangle. If their resultant is a force parallel to BC through the centroid of the triangle, prove that $Q = R = \frac{P}{2}$.

Or

(b) State and prove Varigon's theorem.

18. (a) State and prove two Trigonometical theorems.

Or

(b) A heavy uniform sphere rests touching two smooth inclined planes one of which is inclined at 60° to the horizontal. If the pressure on this plane is one – half of the weight of the sphere, prove that the inclination of the other plane to the horizontal is 30°.

19. (a) A ladder 20 meters long with its centre of gravity 8 meters up from the bottom, weights 60 kg and rests at an angle of θ to the ground against a smooth vertical wall. The coefficient of friction between the ladder and the ground is $\frac{1}{2}$. Find the least value of θ which will enable a weighing 140 kg to reach the top with out the ladder slipping.

Or

(b) A body is at rest on a rough inclined plane and is acted upon by a force parallel to the plane Find the limits between which the force must lie.

20. (a) Find the tension at any point of the catenary.

Or

(b) Explain the parabolic catenary.

———————

(8 pages)

Reg. No. : ....

Code No. : 20066 E **Sub. Code : SMMA 54/ AMMA 54**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fifth Semester

Mathematics — Core

TRANSFORMS AND THEIR APPLICATIONS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $F\{f(x)\} = \bar{f}(s)$, then $F\{e^{iax} f(x)\} =$ ________

(a) $\bar{f}(s+a)$ (b) $\bar{f}(x+a)$

(c) $\bar{f}(s-a)$ (d) $\bar{f}(x-a)$

2. $F_S\{f''(x)\} =$ ________

(a) $-sF_s\{f(x)\} + f(0)$

(b) $-s^2 F_s\{f(x)\} + sf(0)$

(c) $-sF_c\{f(x)\} + f(0)$

(d) $-s^2 F_c\{f(x)\} + sf(0)$

3. If $F\{f(x)\} = \bar{f}(s)$, then $F\{f(ax)\} =$ ________

(a) $\frac{1}{|a|}\bar{f}\left(\frac{s}{a}\right)$ (b) $\bar{f}\left(\frac{s}{a}\right)$

(c) $|a|\bar{f}(sa)$ (d) $\bar{f}(sa)$

4. $\frac{d}{ds}\{F_c(f(x))\} =$ ________

(a) $F_c\{xf(x)\}$ (b) $-F_c\{xf(x)\}$

(c) $F_s\{xf(x)\}$ (d) $-F_s\{xf(x)\}$

5. $F_c\{f(x)\} = \bar{f}_c(n) =$ ________

(a) $\int_0^l f(x)\sin\left(\frac{n\pi .x}{l}\right)dx$

(b) $\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$

(c) $\int_0^l f(x)\sin(n\pi x)dx$

(d) $\int_0^l f(x)\cos(n\pi x)dx$

6. $F_c\{f'(x)\} =$ ________

(a) $\frac{-n\pi}{l}\bar{f}_c(n)$

(b) $(-1)^n f(l) - f(0) + \frac{n\pi}{l}\bar{f}_s(n)$

(c) $\frac{n\pi}{l} f_c(n)$

(d) $(-1)^n f(l) + f(0) - \frac{n\pi}{l}\bar{f}_s(n)$

7. $Z(n) =$ ________, where ROC is $|z| > 1$.

(a) $\frac{z}{(z+1)^2}$ (b) $\frac{z(z+1)}{(z-1)^3}$

(c) $\frac{z}{(z-1)^2}$ (d) $\frac{z(z-1)}{(z+1)^2}$

8. $z(a^n) =$ ________, if $|z| > a$.

(a) $\frac{z}{z-a}$ (b) $\frac{z}{z+a}$

(c) $\frac{nz}{z-a}$ (d) $\frac{nz}{z+a}$

9. $z^{-1}\left\{\frac{1}{z+2}\right\} =$ ________

(a) 2^n (b) $(-2)^n$

(c) $(-2)^{n-1}$ (d) 2^{n-1}

10. $z^{-1}\{e^{a/z}\} =$ ________

(a) a^{n-1} (b) a^n

(c) $\frac{a^{n-1}}{(n-1)!}$ (d) $\frac{a_n}{n!}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the Fourier transform of $f(x)$, defined as $f(x) = \begin{cases} 1, & \text{for } |x| < a \\ 0, & \text{for } |x| > a \end{cases}$ and hence find the value of $\int_0^\infty \frac{\sin x}{x} dx$.

Or

(b) Find the Fourier transform of $\left\{\frac{\sin ax}{x}\right\}$ and hence prove that $\int_{-\infty}^{\infty} \frac{\sin^2 ax}{x^2} = a\pi$.

[P.T.O.]

12. (a) Find the Fourier cosine transform of e^{-ax} and use it to find the Fourièr transform of $e^{-a|x|}\cos bx$.

Or

(b) Find $F_c(e^{-a^2x^2})$.

13. (a) Find the finite Fourier sine and cosine transforms of $\left[1-\frac{x}{\pi}\right]^2$ in $(0, \pi)$.

Or

(b) Find the finite Fourier sine and cosine transforms of e^{ax} in $(0, l)$.

14. (a) Find the z – transform of $t^2 e^{-t}$.

Or

(b) Find the z – transform of $n\cos n\theta$.

15. (a) Find $z^{-1}\left\{\frac{1}{1+4z^{-2}}\right\}$ by the long division method.

Or

(b) Find $z^{-1}\left\{\frac{2z^2+4z}{(z-2)^3}\right\}$, by using Residue theorem.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve the wave equation $\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2}$, subject to the initial conditions $y(x,0) = f(x)$, $-\infty < x < \infty$. $\frac{\partial y}{\partial t}(x, 0) = g(x)$ and the boundary conditions $y(x, t) \to 0$ as $x \to \pm\infty$.

Or

(b) Solve the equation $(D^2 - 4D + 3)y = \cos 3x$, $x > 0$, given that $y(0) = 0$ and $y'(0) = 0$.

17. (a) Find $f(x)$, if its Fourier sine transform is $\left(\frac{s}{s^2+1}\right)$.

Or

(b) Solve the equation $\frac{\partial u}{\partial t} = \alpha^2\frac{\partial^2 u}{\partial x^2}$, satisfying the boundary conditions $\frac{\partial u}{\partial x}(0, t) = k$, $t \geq 0$ and $u(x,t) \to 0$ as $x \to 0$ and the initial condition $u(x, 0) = 0$.

18. (a) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < \pi$, $t > 0$, using finite Fourier transforms, given that $u(0,t) = 0$, $u(\pi, t) = 0$, for $t > 0$ and $u(x, 0) = 4\sin^3 x$.

Or

(b) Solve the equation $\frac{\partial u}{\partial t} = \alpha^2\frac{\partial^2 u}{\partial x^2}$, $0 < x < l$, using finite Fourier transform, given that $\frac{\partial u}{\partial x}(0, t) = 0$, $\frac{\partial u}{\partial x}(l, t) = 0$ for $t > 0$ and $u(x,0) = kx$, for $0 < x < l$.

19. (a) Find the z – transform of the following functions

(i) $r^n\cos n\theta$

(ii) $r^n\sin n\theta$

(iii) $\cos n\theta$

(iv) $\sin n\theta$.

Or

(b) Find z – transform of

(i) $f(n) = \frac{1}{n(n-1)}$ and

(ii) $f(n) = \frac{2n+3}{(n+1)(n+2)}$.

20. (a) Find $z^{-1}\left\{\frac{z^2+2z}{z^2+2z+4}\right\}$ by the method of partial fractions.

Or

(b) Solve the simultaneous difference equations. $x_{n+1} = 7x_n + 10y_n$; $y_{n+1} = x_n + 4y_n$, given that $x_0 = 3$ and $y_0 = 2$.

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Reg. No. : ....

...de No. : 30344 E **Sub. Code : SMMA 61**

(CBCS) DEGREE EXAMINATION, APRIL 2022

Sixth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2017 onwards)

Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer:

f $f(z)=z^2$, then the value of $v(x,y)$

a) x^2-y^2 (b) $2xy$

c) xy (d) x^2+y^2

The complex form of CR equations

(a) $f_x=-if_y$ (b) $f_x=if_y$

(c) $f_y=-if_x$ (d) $f_x=f_y$

3. If C is the circle with center a and radius r, then the value of $\int_C |z'(t)|dt$

(a) $2\pi i$ (b) $-2\pi i$

(c) $2\pi r$ (d) $-2\pi r$

4. If C is the circle $|z-2|=5$, then $\int_C \frac{dz}{z-3}=$ ———

(a) 0 (b) $2\pi i$

(c) $-2\pi i$ (d) πi

5. $z-\frac{z^3}{3!}+\frac{z^5}{5!}-\cdots=?$

(a) $\sin z$ (b) $\cos z$

(c) $\sinh z$ (d) $\cosh z$

6. The poles of $f(z)=\frac{z^2}{(z-2)(z+3)}$

(a) 2, 3 (b) –2, 3

(c) 2, –3 (d) –2, –3

$f(z)=\frac{1}{2z^2+5iz-2}$, then $\text{Res}\{f(z); -i/2\}=?$

) $\frac{1}{3}$ (b) $\frac{1}{3i}$

) $-\frac{1}{3i}$ (d) $-\frac{1}{3}$

he value of $\int_{|z|=2} \tan z\, dz$

) $2\pi i$ (b) $-2\pi i$

) $4\pi i$ (d) $-4\pi i$

he fixed point of the transformation $w=\frac{1}{z-2i}$

) 0 (b) i

) $-i$ (d) $2i$

Which one of the following is not a bilinear transformation

) $w=z$ (b) $w=\overline{z}$

) $w=1+z$ (d) $w=1-z$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $f(z)=\text{Re}\, z$ is nowhere differentiable.

Or

(b) If $f(z)$ and $\overline{f(z)}$ are analytic in a region D show that $f(z)$ is constant in that region.

12. (a) Prove that $\int_{-c} f(z)\,dz=-\int_c f(z)dz$.

Or

(b) Evaluate $\int_C \frac{z\,dz}{z^2-1}$ where C is the positively oriented circle $|z|=2$.

13. (a) Expand $\cos z$ into a Taylor's series about the point $z=\pi/2$.

Or

(b) Find the residue of $\frac{1}{(z^2+a^2)^2}$ at $z=ai$.

14. (a) Evaluate $\int_C \frac{dz}{2z+3}$ where C is $|z| = 2$.

Or

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$.

15. (a) Find the bilinear transformation which maps the point $z = -1, 1, \infty$ respectively on $w = -i, -1, i$.

Or

(b) Find the fixed points of the transformation $w = \frac{1+z}{1-z}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Derive the CR equations in polar co-ordinates.

Or

(b) Find the analytic function $f(z) = u + iv$ given that $u - v = e^x(\cos y - \sin y)$.

17. (a) State and prove Cauchy's integral formula.

Or

(b) State and prove fundamental theorem of algebra.

18. (a) State and prove Maclaurin's series.

Or

(b) State and prove Cauchy's residue theorem.

19. (a) Evaluate $\int_0^{\infty} \frac{x \sin x}{x^2 + a^2} dx$.

Or

(b) Prove that $\int_0^{\infty} \frac{\cos x}{1+x^2} dx = \frac{\pi}{2e}$.

20. (a) Find the points where the following mappings are conformal. Also find the critical points if any (i) $w = z^n$ (ii) $w = \frac{1}{2}$.

Or

(b) Prove that a bilinear transformation preserves inverse points.

———————

(7 pages)

Reg. No. : ....

Code No. : 20067 E **Sub. Code : SMMA 61**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Sixth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The function $f(z)=\bar{z}$ is

(a) differentiable

(b) nowhere differentiable

(c) differentiable only at (0, 0)

(d) none of these

2. If $f(z)=u+iv$ is analytic and $f(z)\neq 0$, then $\nabla^2 amp\ f(z)=$ ————

(a) ∞ (b) 1

(c) $\tan^{-1}\left(\frac{v}{u}\right)$ (d) 0

3. The value of $\int_c \frac{dz}{z-a}$, (c is $|z|=r$) is ————

(a) $2\pi r$ (b) 2π

(c) $2\pi i$ (d) 0

4. The value of $\int_c \frac{dz}{z-3}$, where c is $|z|=2$

(a) 0 (b) $2\pi i$

(c) $6\pi i$ (d) 1

5. $z-\frac{z^2}{2}+\frac{z^3}{3}-\cdots(-1)^{n-1}\frac{z^n}{n}+\cdots$ represents which of following function?

(a) $\frac{1}{1-z}$ (b) $\log(1+z)$

(c) $\log(1-z)$ (d) $\sin z$

6. ———— is the singular point for the function $ze^{1/z}$

(a) $z=\infty$ (b) $z=0$

(c) $z=1$ (d) $z=-\infty$

7. Where we evaluated the $\int_0^{2\pi} f(\cos\theta,\sin\theta)\,d\theta$, type of integrals?

(a) $|z|>r$ (b) $|z|<r$

(c) $|z|=1$ (d) $|z|=r$

8. By Jordan's Lemma, the value of $\lim_{r\to\infty}\int_c f(z)e^{iaz}dz=$ ————, where c is the semi-circle.

(a) r (b) $-\infty$

(c) ∞ (d) 0

9. The critical point of the transformation $w=az+b$ is ————

(a) ± 1 (b) a

(c) 0 (d) no critical points

10. Under the transformation $w=iz+1$, then image of $x>0$ is ————

(a) $v>0$ (b) $u>0$

(c) $-1<u<1$ (d) $v<0$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Show that $f(z)=\sqrt{r}(\cos\theta/2+i\sin\theta/2)$, where $r>0$ and $0<\theta<2\pi$ is differentiable.

Or

(b) If $f(z)=u+iv$ is analytic and $f(z)\neq 0$ then prove that $\nabla^2\log|f(z)|=0$.

12. (a) Evaluate $\int_c \frac{z+2}{z}dz$, where c is the semi circle $z=2e^{i\theta}$, where $0\le\theta\le\pi$.

Or

(b) State and prove Liouville's theorem.

13. (a) Find the Taylor series to represent $\frac{z^2-1}{(z+2)(z+3)}$ in $|z|<2$.

Or

(b) Use Laurent's series to find the residue of $\frac{d^{2z}}{(z-1)^2}$ at $z=1$.

14. (a) Evaluate $\int_0^{2\pi}\frac{d\theta}{5+4\sin\theta}$.

Or

(b) Evaluate $\int_0^{\infty}\frac{dx}{x^2+1}$.

15. (a) Find the image of the circle $|z-3|=5$ under the transformation $w=\frac{1}{z}$.

Or

(b) Prove that a bilinear transformation $w=\frac{az+b}{cz+d}$, where $ad-bc\neq 0$ maps the real axis into itself iff a,b,c,d are real.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) State and prove $C-R$ equations in polar form.

Or

(b) Prove that $u=2x-x^3+3xy^2$ is harmonic and find its harmonic conjugate. Also find the corresponding analytic function.

17. (a) State and prove Cauchy's theorem.

Or

(b) Evaluate $\int_c \frac{e^z}{(z+2)(z+1)^2}$ where c is $|z|=3$.

18. (a) Expand : $f(z)=\frac{z}{(z-1)(z-2)}$ in a Laurent's series valid for (i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$.

Or

(b) Use Cauchy residue theorem, to evaluate $\int_c \frac{3z^2+z-1}{(z^2-1)(z-3)}dz$, around the circle $|z|=2$.

19. (a) Prove that $\int_0^{2\pi}\frac{d\theta}{1+a\sin\theta}=\frac{2\pi}{\sqrt{1-a^2}}$, $(-1<a<1)$.

Or

(b) Evaluate : $\int_{-\infty}^{\infty}\frac{\cos x}{(x^2+a^2)(x^2+b^2)}dx$, $(a>b>0)$.

20. (a) Find the bilinear transformation which maps $-1, 0, 1$ of the z-plane onto $-1, -i, 1$ of the co-plane.

Or

(b) Prove that a bilinear transformation preserves inverse points.

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ıges) **Reg. No. :** ....

ode No. : 30345 E **Sub. Code : SMMA 62**

.c. (CBCS) DEGREE EXAMINATION, APRIL 2022

Sixth Semester

Mathematics — Core

NUMBER THEORY

(For those who joined in July 2017 onwards)

me : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

The value of $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 =$

(a) $\binom{2n+2}{3}$ (b) $\binom{2n}{n}$

(c) $\binom{2n}{3}$ (d) $\binom{2n+1}{3}$

2. If n is an odd integer, and $r = \frac{1}{2}(n-1)$, then

(a) $\binom{n}{r} = \binom{n}{r-1}$ (b) $\binom{n}{r} = \binom{n+1}{r+1}$

(c) $\binom{n}{r} = \binom{n}{r+1}$ (d) $\binom{n+1}{r} = \binom{n+1}{r+1}$

3. $lcm(3054, 12378) =$ ————

(a) 6300402 (b) 3054

(c) 12378 (d) 6

4. Given indegers a,b,c,d, which one of the following is false?

(a) If $a \mid bc$ then $a \mid c$

(b) If $a \mid b$ and $a \mid c$ then $a^2 \mid bc$

(c) $a \mid b$ if and only if $ac \mid bc$, where $c = 0$

(d) If $a \mid b$ and $c \mid d$ then $ac \mid bd$

5. Prime factorization of 17460 is ————

(a) 8.9.5.49 (b) $2^3.3^2.5.7^2$

(c) $2^3.3.5.7^3$ (d) $8.9.5.7^2$

For $n \geq 2$, $\sqrt[n]{n}$ is ————

(a) irrational (b) rational

(c) composite (d) integer

The remainder of $2^{20} - 1$ is divisible by 41 ————

(a) 1 (b) 2

(c) 3 (d) 0

Number of solutions of $18x \equiv 30 \pmod{42}$ is ————.

(a) 2 (b) 6

(c) 3 (d) 5

Any absolute pseudoprime is ————

(a) square free (b) pseudo prime

(c) prime (d) absolute

The unit digit of 3^{100} is ————

(a) 0 (b) 1

(c) 2 (d) 3

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $1.2 + 2.3 + 3.4 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$, for all $n \geq 1$.

Or

(b) Derive the Binomial identity $\binom{2}{2} + \binom{4}{2} + \binom{6}{2} + \cdots + \binom{2n}{2} = \frac{n(n+1)(4n-1)}{6}$ $n \geq 2$.

12. (a) Show that the expression $a(a^2+2)/3$ is an integer for all $a \geq 1$.

Or

(b) For any integers a,b prove that if $a \mid b$ and $b \neq 0$ then $|a| \leq |b|$.

13. (a) State and prove Euclid's theorem.

Or

(b) Employing the Sieve of eratosthenes, obtain all the primes between 100 and 200.

[P.T.O]

14. (a) If $ca = cb(\text{mod}\, n)$ then prove that $a \equiv b(\text{mod}\, n/d)$, where $d = \gcd(c,n)$.

Or

(b) Find the remainder when $1! + 2! + \cdots + 100!$ is divided by 12.

15. (a) State and prove Wilson's theorem.

Or

(b) If p and q are distinct primes with $a^p \equiv a(\text{mod}\, p)$ and $a^q \equiv a(\text{mod}\, q)$ then prove that $a^{pq} \equiv a(\text{mod}\, pq)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove first principle of induction.

Or

(b) Prove that the sum of the reciprocals of the first 'n' triangular numbers is less than 2.

17. (a) State and prove division algorithm.

Or

(b) Find the solution of linear diophantine equation $24x + 138y = 18$.

18. (a) If all the $n > 2$ terms of the arithmetic progression $p, p+d, \ldots p+(n-1)d$ are prime numbers then prove that the common difference d is divisible by every prime $q < n$.

Or

(b) State and prove Fundamental theorem of Arithmetic.

19. (a) State and prove Chineese remainder theorem.

Or

(b) Find the solutions of the system of congruences $3x + 4y \equiv 5(\text{mod}\, 13)$, $2x + 5y \equiv 7(\text{mod}\, 13)$.

20. (a) State and prove Fermat's theorem.

Or

(b) Prove that the quadratic congruence $x^2 + 1 \equiv 0(\text{mod}\, p)$, where p is an odd prime has a solution iff $p \equiv 1(\text{mod}\, 4)$.

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(6 pages)

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B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Sixth Semester

Mathematics — Core

NUMBER THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The value of $\binom{n}{0}$ is ————
   (a) 0 (b) n
   (c) 1 (d) $n!$

2. Who say "Everything is number"? ————
   (a) Pythagoreans (b) Egyptians
   (c) Greacean (d) Babylonians

3. gcd (8, 17) is ————
   (a) 2 (b) 8
   (c) 1 (d) 0

4. If lcm $(a,b) = ab$, then gcd (a,b) is
   (a) 0 (b) 1
   (c) ab (d) $(ab)^2$

5. The value of $\pi_{4,3}(89)$ is ————
   (a) 4 (b) 3
   (c) 10 (d) 13

6. Which of the following is twin prime numbers?
   (a) 2, 3 (b) 5, 7
   (c) 19, 23 (d) 79, 97

7. If $100x \equiv 0 (\text{mod } 3)$, then the value of x is ————
   (a) 1 (b) 2
   (c) 3 (d) 4

8. Which one of the following is correct? ————
   (a) $7 \equiv 0(\text{mod } 5)$ (b) $7 \equiv 0(\text{mod } 6)$
   (c) $7 \equiv 0(\text{mod } 7)$ (d) $7 \equiv 0(\text{mod } 8)$

9. Which one is the smallest pseudoprime to base 3
   (a) 91 (b) 217
   (c) 341 (d) 561

10. Fermat's theorem says ————
    (a) $a^{p-1} \equiv 0(\text{mod } p)$
    (b) $a^{p-1} \equiv 1(\text{mod } p)$
    (c) $a^{p-1} \equiv -1(\text{mod } p)$
    (d) $a^{p} \equiv p-1(\text{mod } p)$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) State and prove Archimedean property.

    Or

    (b) Prove that $\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$, $1 \le k \le n$.

12. (a) If $a \mid c$ and $b \mid c$ with gcd $(a,b) = 1$, then prove that $ab \mid c$.

    Or

    (b) Prove that any positive integers a and b, gcd (a,b) lcm $(a,b) = ab$.

13. (a) Prove that the number $\sqrt{2}$ is irrational.

    Or

    (b) Prove that there are an infinite number of primes of the form $4n+3$.

14. (a) Prove that 41 divides $2^{20} - 1$.

    Or

    (b) Solve $9x \equiv 21(\text{mod } 30)$.

15. (a) State and prove Fermat's theorem.

    Or

    (b) Using Fermat's method to factorize the number 119143.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Prove that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

using induction.

Or

(b) State and prove binomial theorem.

17. (a) State and prove division algorithm.

Or

(b) Solve the Diophantine equation $24x + 138y = 18$.

18. (a) State and prove fundamental theorem of Arithmetic.

Or

(b) If p_n is the n^{th} prime, then prove that $p_n \le 2^{2^{n-1}}$.

19. (a) State and prove Chinese remainder theorem.

Or

(b) Explain the Basic properties of congruence.

20. (a) State and prove Wilson's theorem.

Or

(b) Prove that if n is an odd pseudoprime, then $M_n = 2^n - 1$ is a larger one.

(7 pages)

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Code No. : 20069 E **Sub. Code : SMMA 63**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Sixth Semester

Mathematics — Core

GRAPH THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The maximum degree of a point in a graph with p points is ————

   (a) p (b) $p-1$

   (c) q (d) p^2

2. If G is a (p,q) graph, then ————

   (a) $q \le \binom{p}{2}$ (b) $q = \binom{p}{2}$

   (c) $q \ge \binom{p}{2}$ (d) $q = p-1$

3. Which of the following is a graphic sequence?

   (a) (1, 1, 1) (b) (2, 2, 1)

   (c) (2, 1, 1) (d) (1, 0, 0)

4. The connectivity of the complete graph K_p is ————

   (a) p (b) 0

   (c) 1 (d) $p-1$

5. Which of the following is an Eulerian graph?

   (a) K_6 (b) K_7

   (c) $K_{3,3}$ (d) $K_{2,5}$

6. Every Hamiltonian graph is ———— connected.

   (a) 2 (b) p

   (c) $p-1$ (d) q

7. If G is a maximal planar (p,q) graph, then ————

   (a) $q \le 2p-4$ (b) $q \le 3p-6$

   (c) $q \ge 3p-6$ (d) $q = 3p-6$

8. $\chi(K_{2,10}) =$

   (a) 10 (b) 20

   (c) 2 (d) 12

9. If G is a (p,q) graph, then the coefficient of λ^{p-1} in $f(G,\lambda)$ is ————

   (a) 0 (b) q

   (c) $-q$ (d) p

10. If a complete digraph has n vertices, then it has ———— arcs.

    (a) $n(n-1)$ (b) $\frac{n(n-1)}{2}$

    (c) $n-1$ (d) $n(n+1)$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Prove : $\alpha' + \beta' = p$.

    Or

    (b) Prove that any self complementary graph has $4n$ or $4n+1$ points.

12. (a) Verify whether the partition (4, 4, 4, 2, 2, 2) is graphical. If it is graphical, draw the corresponding graph.

    Or

    (b) Prove : A line x of a connected graph G is a bridge if and only if x is not on any cycle of G.

13. (a) If G is a graph with $p \ge 3$ and $\delta \ge \frac{p}{2}$, then show that G is Hamiltonian.

    Or

    (b) Prove that every tree has a center consisting of either one point or two adjacent points.

[P.T.O.]

14. (a) State prove that Euler's theorem on a connected plane graph.

Or

(b) Show that every uniquely n-colourable graph is $(n-1)$-connected.

15. (a) Prove that $\lambda^4 - 3\lambda^3 + 3\lambda^2$ cannot be the chromatic polynomial of any graph.

Or

(b) Define :

(i) Strongly connected digraph

(ii) Unilaterally connected digraph.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 600 words.

16. (a) Show that the maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4}\right]$.

Or

(b) Let G_1 be a (p_1, q_1) graph and G_2 be a (p_2, q_2) graph. Then prove :

(i) $G_1 \times G_2$ is a (p_1p_2, q_1p_2, q_2p_1) graph

(ii) $G_1[G_2]$ is a $(p_1p_2, p_1q_2, p_2^2q_1)$ graph.

17. (a) State and prove a necessary and sufficient condition for a partition $P = (d_1, d_2, ..., d_p)$ of an even number into parts with $p-1 \geq d_1 \geq d_2 \geq \cdots d_p$ to be graphical.

Or

(b) Show that a graph G with at least two points is bipartite if and only if all its cycles are of even length.

18. (a) Prove that $c(G)$ is well define.

Or

(b) Let G be a (p,q)- graph. Prove that the following are equivalent :

(i) G is a tree

(ii) Every two points of G are joined by a unique path

(iii) G is connected and $p = q+1$

(iv) G is acyclic and $p = q+1$.

19. (a) Prove : $\chi'(K_n) = n$, if $n \neq 1$ is odd

$= n-1$, if n is even.

Or

(b) Show that K_5 and $K_{3,3}$ are non planar graphs.

20. (a) If G is a tree with $n \geq 2$ points, then show that $f(G,\lambda) = \lambda(\lambda-1)^{n-1}$.

Or

(b) Prove that the edges of connected graph G can be oriented so that the regulating digraph is strongly connected if and only if every edge of G is contained in at least one cycle.

———————

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Sc. (CBCS) DEGREE EXAMINATION, APRIL 2022

Sixth Semester

Mathematics — Main

DYNAMICS

(For those who joined in July 2017 onwards)

ne : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

The horizontal range R of a projectile is ————

(a) $\frac{u^2 \sin\alpha}{g}$ (b) $\frac{u^2}{g}$

(c) $\frac{u^2 \sin 2\alpha}{g}$ (d) None

2. A particle is projected with a velocity of 24 m/sec at an elevation of 30°. Then the time of flight is

(a) $\frac{12}{g}$ (b) $\frac{36}{g}$

(c) $\frac{24^2}{g}$ (d) $\frac{24}{g}$

3. When two equal sphere impinge directly, their velocities are inter changed. Then their elasticity is ————

(a) 0 (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) 1

4. When an elastic sphere strikes a plane normally with velocity u, it rebounds in the same direction with velocity ————

(a) u (b) eu

(c) u/e (d) e^2u

5. The period of SHM $x = a\cos 2t + b\sin 2t$ is ————

(a) 3π (b) 2π

(c) $\pi/2$ (d) π

In a simple Harmonic motion, the phase at time t is ————

(a) $t + \frac{E}{\sqrt{\mu}}$ (b) $t + \frac{1}{\sqrt{\mu}}$

(c) $t - \frac{E}{\sqrt{\mu}}$ (d) $t + \frac{1}{2E}$

The acceleration of a particle describing a circle of radius a has the component ———— along the radius to the center.

(a) $a\dot{\theta}^2$ (b) $a\theta$

(c) $a\ddot{\theta}$ (d) $a\ddot{\theta}^2$

The transverse component of velocity is ————

(a) $r\theta$ (b) $\dot{r}$

(c) $r\dot{\theta}$ (d) $\ddot{r} - r\dot{\theta}^2$

The pedal equation of the parabola – pole at focus is ————

(a) $r^2 = 2ap$ (b) $r^2 = ap$

(c) $p^2 = 2ar$ (d) $p^2 = ar$

10. The $(p - r)$ equation of the central orbit is ————

(a) $h\big/p^2 \frac{dp}{dr} = F$ (b) $h^2\big/p \frac{dp}{dr} = F$

(c) $h^2\big/p^2 \frac{dp}{dr} = F$ (d) $h^2\big/p^3 \frac{dp}{dr} = F$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Find the range of a particle projected on an inclined plane.

Or

(b) A revolver can fire a bullet with a velocity of 63 m per sec. Is it possible to hit the top of a tower 400 m away its height being 30 m?

12. (a) Find the velocities of two smooth spheres after their direct impact.

Or

(b) A ball overtakes another ball of m times its mass, which is moving with $\frac{1}{n}$ th of its velocity in the same direction. If the impact reduces the first ball to rest, prove that the coefficient of elasticity is $\frac{m+n}{m(n-1)}$.

[P.T.O]

13. (a) Find the composition of two simple harmonic motions of the same period in two perpendicular directions.

Or

(b) Show that the energy of a system executing SHM is proportional to the square of the amplitude and of the frequency.

14. (a) Find the velocity and acceleration in polar co-ordinates.

Or

(b) Find the polar equation of equiangular spiral.

15. (a) Find the law of force towards the pole under which the curve $r^n = a^n \cos n\theta$ can be obtained.

Or

(b) Derive the pedal equation for hyperbola – pole at focus.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Show that the greatest height which a particle with initial velocity v can reach on a vertical wall at a distance 'a' from the point of projection is $\frac{v^2}{2g} - \frac{ga^2}{2v^2}$.

Or

(b) Show that the path of projectile is parabola.

17. (a) Find the loss of kinetic energy due to direct impact between two smooth spheres.

Or

(b) A particle is projected from a point on an inclined plane and at the r^{th} impact it strikes the plane perpendicularly and at the n^{th} impact is at the point of projection. Show that $e^n - 2e^r + 1 = 0$.

18. (a) Write the fundamental differential equation of a particle in simple harmonic motion. Solve it completely.

Or

(b) If the displacement of a moving point at any time be given by an equation of the form $x = a\cos wt + b\sin wt$. Show that the motion is a simple harmonic motion. If $a = 3$, $b = 4$, $w = 2$, determine the period, amplitude, maximum velocity.

19. (a) Show that the path of a point P which possess two constant velocities u and v, the first of which is in a fixed direction and the second of which is perpendicular to the radius OP drawn from a fixed point O is a conic whose focus is O and eccentricity is u/v.

Or

(b) A point describes a curve with constant velocity and its angular velocity about the given fixed point O varies inversely as the distance from O, show that the curve is an equiangular spiral whole pole is O and that the acceleration of the point is along the normal at P and varies inversely as OP.

20. (a) Derive the differential equation of a central orbit in polar co-ordinates.

Or

(b) A particle moves in an ellipse under a force which is always directed towards its focus. Find the law of force, the velocity at any point of the path and its periodic time.

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B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Sixth Semester

Mathematics — Core

DYNAMICS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Time taken by the projectile to reach the greatest height is ————

   (a) $\frac{u\sin\alpha}{g}$ (b) $\frac{u^2\sin\alpha}{g}$

   (c) $\frac{u\sin 2\alpha}{g}$ (d) $\frac{u\sin\alpha}{g^2}$

2. The time of flight of a projectile on an inclined plane is ————

   (a) $\frac{u\sin(\alpha-\beta)}{g}$ (b) $\frac{2u\sin\alpha}{g}$

   (c) $\frac{2u\sin(\alpha-\beta)}{g}$ (d) $\frac{2u\sin(\alpha-\beta)}{g\cos\beta}$

3. The ball is inelastic if

   (a) $v=u$ (b) $v=0$

   (c) $u=\sin\alpha$ (d) $v=1$

4. In Newton's experimental law, the value of e always lies between

   (a) 1 and 2 (b) −1 and 1

   (c) 0 and 1 (d) none of these

5. The equation of simple harmonic motion is

   (a) $\frac{d^2x}{dt^2}=\mu x$ (b) $\frac{d^2x}{dt^2}=-x$

   (c) $\frac{d^2x}{dt^2}=x$ (d) $\frac{d^2x}{dt^2}=-\mu x$

6. The frequency is the reciprocal of ————

   (a) amplitude (b) displacement

   (c) period (d) none

7. The transverse component of velocity is ————

   (a) $\dot{r}$ (b) $r\theta$

   (c) $r\dot{\theta}$ (d) $\dot{r}\dot{\theta}$

8. The polar equation of the equiangular spiral

   (a) $r=e^{\cot\alpha}$ (b) $r=ae^{\cot\alpha}$

   (c) $r=ae^{\theta\cot\alpha}$ (d) $r=\cot\alpha$

9. $(p-r)$ equation of the equiangular spiral is

   (a) $p=\sin\alpha$ (b) $p=r\sin\alpha$

   (c) $p=r$ (d) $p=\cos\alpha$

10. $(p-r)$ equation of the parabola is

    (a) $p=ar$ (b) $p=ar^2$

    (c) $p^2=ar$ (d) $p^2=ar^2$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Define a projectile and derive the greatest height attained by it.

    Or

    (b) Derive the range on an inclined plane.

12. (a) Explain the impact of a smooth sphere on a fixed smooth plane.

    Or

    (b) Explain the oblique impact of two smooth spheres.

13. (a) Define simple harmonic motion and derive the equation of motion.

    Or

    (b) Derive the general solution of the simple harmonic motion equation.

14. (a) Explain the equiangular spiral.

    Or

    (b) Derive the radial component of acceleration.

15. (a) Explain the $(p-r)$ equation of the circle.

    Or

    (b) Explain the velocities in a central orbit.

[P.T.O.]

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Prove that the path of a projectile is a parabola.

Or

(b) Find the greatest distance of the projectile from the inclined plane and show that is attained in half the total time of flight.

17. (a) Find the loss of kinetic energy due to direct impact of two smooth spheres.

Or

(b) Explain the Newton's experimental law.

18. (a) Find the differential equation of a SHM.

Or

(b) Explain the geometrical representation of simple harmonic motion.

19. (a) Explain the velocity and acceleration in plan coordinates.

Or

(b) Find the differential equation of central orbits.

20. (a) Find the pedal equation of the central orbit.

Or

(b) Find the law of force to an internal point under which a body will describe a circle.

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. (CBCS) DEGREE EXAMINATION, APRIL 2022

Sixth Semester

Mathematics — Core

NUMERICAL METHODS

(For those who joined in July 2017 onwards)

e : Three hours   Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

Order of convergence of Newton's method is

(a) 2   (b) 3

(c) 4   (d) 1

In Gauss – Elimination method the coefficient matrix is converted to a ————

(a) Triangular matrix

(b) Upper triangular matrix

(c) Lower triangular matrix

(d) None of these

3. $\Delta =$ ————

(a) $E-1$ (b) $1-E$

(c) $1+E$ (d) $E^{-1}+1$

4. $\Delta y_0 =$

(a) $y_0 - y_1$ (b) $y_1 - y_0$

(c) $y_2 - y_0$ (d) $y_0 - y_2$

5. In the Gauss forward interpolation formula the value of u is

(a) $\frac{x-x_0}{h!}$ (b) $\frac{x-x_0}{h}$

(c) $\frac{x_0-x}{h}$ (d) $\frac{x+x_0}{h}$

6. The Gauss backward formula involves odd differences ———— the central line.

(a) above   (b) on

(c) below or above   (d) below

The accuracy of the trapezoidal rule can be improved by ————

(a) increasing the number of intervals

(b) increasing the value of h

(c) decreasing the number of intervals

(d) none of these

The error in Simpson's one third rule is of order

(a) h^4 (b) h^5

(c) h^2 (d) linear

The order of $\Delta^2 u_x - 5\Delta u_x - 7u_x = 0$ is ————

(a) 3   (b) 2

(c) 1   (d) 0

The degree of $y_x y_{x+1}^2 - y_{x+2} y_x + 5y_x = x^2 + 7$ is ————

(a) 2   (b) 1

(c) 3   (d) none of these

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve the following system of equations by Gauss Jacobi method.

$8x - 3y + 2z = 20$, $4x + 11y - z = 33$, $6x + 3y + 12z = 35$.

Or

(b) Find an iterative formula to find $\sqrt{N}$ where N is a +ve.

12. (a) Find the sixth term of the sequence 8, 12, 19, 29, 42,...

Or

(b) Evaluate $\Delta^n (e^{ax} + b)$.

13. (a) Using the following table, apply Gauss's forward formula to get $f(3.75)$.

x :	2.5	3.0	3.5	4.0	4.5	5.0
$f(x)$	24.145	22.043	20.225	18.644	17.264	16.047

Or

(b) The following table gives some relation between steam pressure and temperature. Find the pressure at temperature 372.1.

T:	361°	367°	378°	387°	399°
P:	154.9	167.9	191.0	212.5	244.2

14. (a) Find $\frac{dy}{dx}$ at the midpoint of

x:	0	300	600	900	1200	1500	1800
y:	135	149	157	183	201	205	193

Or

(b) Evaluate the integral $I = \int_4^{5.2} \log_e x dx$ using Simpson's rule.

15. (a) Form the difference equation given by $y_n (An + B)3^n$.

Or

(b) Solve $y_{n+1} = \sqrt{y_n}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find the approximate root of $x \log_{10} x - 1.2 = 0$ by False position method.

Or

(b) Find the positive root of $f(x) = 2x^3 - 3x - 6 = 0$ by Newton Raphson method correct to 5 decimal places.

17. (a) Prove that

(i) $E\nabla = \Delta = \nabla E$

(ii) $E^{1/2} = \mu + 1/2\delta$

(iii) $\nabla\Delta = \Delta - \nabla = \delta^2$

(iv) $\delta E^{1/2} = \Delta$.

Or

(b) Estimate the production for 1964 and 1966 from the following data.

Year:	1961	1962	1963	1964
Production:	200	220	260	–
Year:	1965	1966	1967	
Production:	350	–	430	

18. (a) From the following table estimate $e^{0.644}$ correct to five decimals using Bessel's formulate. Also find e^x at $x = 0.638$.

x:	0.61	0.62	0.63	0.64
y:	1.840431	1.858928	1.877610	1.896481
x:	0.65	0.66	0.67	
y:	1.915541	1.934792	1.954237	

Or

(b) Use Lagrange's formula to fit a polynomial to the data.

x:	–1	0	2	3
y:	–8	3	1	12

and hence find $y(x = 1)$.

19. (a) Given the following data, find $y'(6)$ and the maximum value of y.

x :	0	2	3	4	7	9
y :	4	26	58	112	466	922

Or

(b) By dividing the range into ten equal parts evaluate $\int_0^{\pi} \sin x dx$ by Trapezoidal and Simpson's rule. Verify your answer with integration.

20. (a) Form the Fibanacci difference equation and solve it.

Or

(b) Solve $y_{x+2} - 5y_{x+1} + 6y_x = x^2 + x + 1$.

———————

(8 pages)

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B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Sixth Semester

Mathematics — Core

NUMERICAL METHODS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The positive root of the equation $x^3-x-1=0$ lies between

   (a) 1 and 2 (b) 0 and 1

   (c) 2 and 3 (d) 3 and 4

2. The order of convergence in Newton - Raphson method is ————

   (a) 3 (b) 2

   (c) 1 (d) 4

3. The value of $\Delta(3^x)$ is ————

   (a) 3^x (b) 3^{x+h}

   (c) $3^x(3^h-1)$ (d) 3^x-1

4. $\Delta(\tan^{-1}x)=$ ————

   (a) $\tan^{-1}\left(\frac{h^2}{1+hx+x^2}\right)$

   (b) $\tan^{-1}\left(\frac{h}{1-hx+x^2}\right)$

   (c) $\tan^{-1}\left(\frac{h}{1+hx-x^2}\right)$

   (d) $\tan^{-1}\left(\frac{h}{1+hx+x^2}\right)$

5. From the following data

x :	5	15	22
y :	7	36	160

   $\Delta\, y_7 =$ ————

   (a) 3.0 (b) 3.1

   (c) 2.9 (d) 2.8

6. When ————, Stirling's formula is used.

   (a) $-\frac{1}{2}<p<\frac{1}{2}$ (b) $p>\frac{1}{2}$

   (c) $p>-\frac{1}{2}$ (d) $-1<p<1$

7. The order of the error in Simpson's $\frac{1}{3}$ rule is ————

   (a) h (b) h^2

   (c) h^3 (d) h^4

8. If $f(0)=1, f(1/3)=0.75, f(2/3)=0.6, f(1)=0.5$, then the value of $\int_0^1 f(x)dx$ using Trapezoidal rule is

   (a) 0.7 (b) 0.6

   (c) 0.8 (d) 1.6

9. The particular integral of $y_{K+2}-5y_{K-1}+6y_K=6^K$ is ————

   (a) 6^{K+1} (b) 6^{K-1}

   (c) $\frac{6^{K-1}}{2}$ (d) $\frac{6^{K+1}}{2}$

10. The order and degree of the equation $y_{x+2}-3y_{x+1}+5y_x=x^2$ are ————

    (a) 2, 3 (b) 3, 2

    (c) 3, 3 (d) 2, 2

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Find a real root of the equation $\cos x = 3x-1$ correct to 3 decimal places by using iteration method.

    Or

    (b) Determine the root of $xe^x-3=0$ correct to three decimal places using the method of false position.

12. (a) Evaluate $\Delta^{10}\left[(1-x)(1-2x^2)(1-3x^3)(1-4x^4)\right]$ if the interval of differencing is 2.

    Or

    (b) Represent the function $x^4-12x^3+42x^2-30x+9$ and its successive differences in factorial notation where the differencing interval $h=1$.

13. (a) The following data gives the melting point of an alloy of lead and zinc, where t is the temperature in degree centigrade and p is the percentage of lead in the alloy.

p	40	50	60	70	80	90
t	184	204	226	250	276	304

Using Newton's interpolation formula, find the melting point of the alloy containing 84 percentage of lead.

Or

(b) In the table below, estimate the missing value

x	0	1	2	3	4
y	1	2	4	–	16

Explain why it differs from $2^3 = 8$.

14. (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.25$ from the following data.

x	1.00	1.05	1.10	1.15
y	1.00000	1.02470	1.04881	1.07238
x	1.20	1.25	1.30	
y	1.09544	1.11803	1.4017	

Or

(b) Dividing the range into 10 equal parts, find the approximate value of $\int_0^{\pi} \sin x\,dx$ by Simpson's $\frac{1}{3}$rd rule.

15. (a) Solve the difference equation $y_{n+1} - 2y_n \cos\alpha + y_{n-1} = 0$.

Or

(b) Eliminate the constants from $y_n = A.2^n + B.3^n$ and derive the corresponding difference equation of the lowest possible order.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Find the inverse of the matrix $\begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix}$ by using Gaussian elimination method.

Or

(b) Find the negative root of the equation $x^3 - 2x + 5 = 0$.

17. (a) Prove that

$$y_k = y_0 + k\Delta y_0 + \frac{k(k-1)}{1.2}\Delta^2 y_0 + \cdots + \Delta^k y_0.$$

Or

(b) Find the second difference of the polynomial $7x^4 + 12x^3 - 6x^2 + 5x - 3$ with interval of differencing $h = 2$.

18. (a) Prove that $y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(P+2)}{3!}\nabla^3 y_n + \cdots + \frac{p(p+1)\cdots(p+n-1)}{n!}\nabla^n y_n$.

Or

(b)

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

From this table, find the value of $f(8)$ by using Newton's divided difference formula.

19. (a) Find the Newton's backward difference formula to compute the derivatives.

Or

(b) Using the following data, find $f'(5)$

x	0	2	3	4	7	9
$f(x)$	4	26	58	112	466	922

20. (a) Solve the equation

$$y_{n+2} + 2y_{n+1} - 56y_n = 2^n(n^2 - 3).$$

Or

(b) Solve the difference equation $u(x+2) - 4u(x) = 9x^2$.

———————

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Reg. No. : ....

ode No. : 30357 E **Sub. Code : SNMA 4 A/ ANMA 41**

(CBCS) DEGREE EXAMINATION, APRIL 2022

Fourth Semester

Mathematics

Non Major Elective — MATHEMATICS FOR COMPETITIVE EXAMINATIONS — II

(For those who joined in July 2017 onwards)

me : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

The money borrowed for a certain period is called ————

(a) principal (b) simple interest

(c) amount (d) interest

2. How much simple interest will Rs. 2000 earn in 18 months at 6% p.a.?

(a) Rs. 120 (b) Rs. 180

(c) Rs. 216 (d) Rs. 240

3. If A's 1 day work $= \frac{1}{n}$, then A can finish the work in ———— days.

(a) 1 (b) 10

(c) n (d) $-n$

4. If A's 1 day work is $\frac{1}{8}$ and B's 1 day work is $\frac{1}{10}$, then $(A+B)$'s 1 day work is ————

(a) 8 (b) 10

(c) $\frac{40}{9}$ (d) $\frac{9}{40}$

5. x Km/hr = ———— m/sec.

(a) $x \times \frac{18}{5}$ (b) $x \times 18$

(c) $x \times \frac{5}{18}$ (d) $x \times 5$

A train is moving with a speed of 180 km/hr. Its speed is

(a) 5 m/sec (b) 30 m/sec

(c) 40 m/sec (d) 50 m/sec

If 15 dolls cost Rs. 35, what do 39 dolls cost?

(a) Rs. 90 (b) Rs. 91

(c) Rs. 89 (d) Rs. 80

The method of finding the 4th proportion when the other three are given is called ———— proportion.

(a) simple (b) compound

(c) direct (d) indirect

A pipe connected with a reservoir, emptying it is known as ————

(a) inlet (b) outlet

(c) full (d) partly full

10. If a pipe can fill a tank in x hours, then part filled in 1 hr = ————

(a) x (b) 0

(c) $\frac{1}{x}$ (d) n

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) At what rate percent per annum will a sum of money double in 8 years?

Or

(b) Find compound interest on Rs. 10000 at 10% p.a. for 2 years and 3 months, compounded annually.

12. (a) A can build a wall in 30 days, while B alone can built it in 40 days. If they build it together and get a payment of Rs. 7000, what is B's share?

Or

(b) A and B together can complete a piece of work in 12 days, B and C can do it in 20 days and C and A can do it in 15 days. A, B and C together can complete it in how many days?

13. (a) Walking at 7/8 of its usual speed, a train is 10 minutes too late. Find its usual time to cover the journey.

Or

(b) Tow trains approach each other at 30 km/hr and 27 km/hr from two places 342 km apart. After how many hours will they meet?

14. (a) If 20 men can build a wall 112m long in 6 days, what length of a similar wall can be built by 25 men in 3 days?

Or

(b) 5 men or 9 women can do a piece of work in 19 days. In how many days will 3 men and 6 women do it?

15. (a) Two pipes A and B can fill a tank in 24 hours and 30 hours respectively. If both the pipes are opened simultaneously in the empty tank, how much time will be taken by them to fill it.

Or

(b) A pipe can empty a tank in 40 minutes. A second pipe with diameter twice as much as that of the first is also attached with the tank to empty it. The two together can empty the tank in how much time?

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) What annual instalment will discharge a debt of Rs. 4600 due in 4 years at 10% p.a. simple interest?

Or

(b) The difference between compound interest and simple interest on a certain sum at 8% p.a. for 2 years is Rs. 240. Find the sum.

17. (a) A and B can do a piece of work in 12 days; B and C can do it in 15 days while C and A can do it in 20 days. In how many days will they finish it working together? Also, in how many days can A alone do it?

Or

(b) A can do a piece of work in 10 days while B alone can do it in15 days. They work together for 5 days and the rest of the work is done by C in 2 days and the rest of the work is done by C in 2 days. If they get Rs. 4500 for the whole work, how should they divide the money?

18. (a) A and B are two stations 390 km apart. A train starts form A at 10 a.m. and travels towards B at 65 kmph. Another train starts from B at 11 a.m. and travels towards A at 35 kmph. At what time do they meet?

Or

(b) A train traveled distances of 10 km, 20 km and 30 km. At speeds of 50 km/hr, 60 km/hr and 90 km/hr respectively. What is the average speed of the train?

19. (a) If 8 men working 9 hours a day can build a wall 18 m long 2 m broad and 12 m high in 10 days, how many men will be required to build a wall 32 m long, 3 m broad and 9 m high by working 6 hours a day, in 8 days.

Or

(b) 8 women can complete a work in 10 days and 10 children take 16 days to complete the same work. How many days will 10 women and 12 children take to complete the work?

20. (a) A tap can fill the tank in 6 hrs. After half the tank is filled, three more similar taps are opened. What is the total time taken to filled the tank completely?

Or

(b) Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it took 32 minute more to fill the cistern When the cistern is full, in what time will the leak empty it?

———————

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Reg. No. : ....

Code No. : 20077 E **Sub. Code : SSMA 4 A/ ASMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fourth Semester

Mathematics

Skill Based Subject — TRIGONOMETRY, LAPLACE TRANSFORM AND FOURIER SERIES

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The value of $(\cos\theta + i\sin\theta)^n$ is ————
   (a) 0 (b) 1
   (c) $\cos n\theta + i\sin n\theta$ (d) $\sin n\theta + i\cos n\theta$

2. If $\cos\theta + i\sin\theta = x$, then the value of $x + \frac{1}{x}$ is ————
   (a) $2\cos\theta$ (b) $2i\sin\theta$
   (c) $2i\cos\theta$ (d) $2\sin\theta$

3. The value of $2\sinh x\cosh x$ is ————
   (a) 0 (b) 1
   (c) $\cosh 2x$ (d) $\sinh 2x$

4. The value of $\log_e\left(x + \sqrt{x^2+1}\right)$ is ————
   (a) $\sinh x$ (b) $\cosh x$
   (c) $\sinh^{-1} x$ (d) $\cosh^{-1} x$

5. The value of $L(1)$ is ————
   (a) $\frac{1}{s}$ (b) $\frac{1}{s^2}$
   (c) $\frac{2}{s^3}$ (d) 0

6. The value of $L^{-1}\left[\frac{1}{(s+a)^2}\right]$ is ————
   (a) e^{-at} (b) $e^{-at}t$
   (c) $e^{at}t$ (d) e^{at}

7. The value of $L(te^{-at})$ ————

(a) $\frac{1}{s+a}$ (b) $\frac{1}{(s+a)^2}$

(c) $\frac{s}{s+a}$ (d) $\frac{s}{(s+a)^2}$

8. The value of $L^{-1}\left[\frac{2}{(s-a)^3}\right]$ ————

(a) te^{at} (b) t^2e^{at}

(c) te^t (d) t^2e^t

9. The function $\tan x$ is periodic with period ————

(a) 0 (b) 2π

(c) π (d) 3π

10. Which one of the following is an even function?

(a) x (b) x^3

(c) $\sin x$ (d) $e^x + e^{-x}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Express $\frac{\sin 6\theta}{\sin\theta}$ in terms of $\cos\theta$.

Or

(b) Expand $\cos^6\theta$ in series of cosines of multiples of θ.

12. (a) Prove that $\sinh^{-1}x = \log_e\left(x+\sqrt{x^2+1}\right)$.

Or

(b) Find $\log(1-i)$.

13. (a) Find $L(\sin^2 2t)$.

Or

(b) Find $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$.

14. (a) Evaluate $\int_0^\infty e^{-2t}\sin 3t\,dt$.

Or

(b) Find $L^{-1}\left[\frac{1+2s}{(s+2)^2(s-1)^2}\right]$.

15. (a) Express $f(x) = x$ as Fourier series in $-\pi < x < \pi$.

Or

(b) Obtain the half range sine series for e^x in $[0, 1]$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 600 words.

16. (a) Prove that $\frac{ah}{\cos\theta} - \frac{bk}{\sin\theta} = a^2 - b^2$.

Or

(b) Show that

$\sin^3\theta\cos^5\theta =$

$-\frac{1}{2^7(\sin 8\theta + 2\sinh 6\theta 0 2\sin 4\theta - 6\sin 2\theta)}$

17. (a) If $\cosh u = \sec\theta$, show that $u = \log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$.

Or

(b) Find the general value of $\log_{(-3)}(-2)$.

18. (a) Find (i) $L(\cos at)$ (ii) $L(\sinh at)$.

Or

(b) Prove that $L^{-1}\left[\log\frac{s+1}{s-1}\right] = \frac{2\sinh t}{t}$.

19. (a) Solve the equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t$ given that $y = \frac{dy}{dt} = 0$ when $t = 0$.

Or

(b) Solve the equations $3\frac{dx}{dt} + \frac{dy}{dt} + 2x = 1$, $\frac{dx}{dt} + 4\frac{dy}{dt} + 3y = 0$ given $x = 0 = y$ at $t = 0$.

20. (a) Explain the Fourier series for odd and even functions.

Or

(b) Prove that $|\sin x| = \frac{2}{\pi} - \frac{4}{\pi}\left(\cos\frac{2x}{3} + \frac{\cos 4x}{15} + \cdots\right)$.

———————

(8 pages) **Reg. No. :** ....

Code No. : 6837 **Sub. Code : PMAM 22**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Second Semester

Mathematics — Core

ANALYSIS — II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. If $f \in \mathcal{R}(\alpha)$ and $g \in \mathcal{R}(\alpha)$ on $[a, b]$ then $fg \in$

   (a) $\mathcal{R}^2(\alpha)$

   (b) $\mathcal{R}(\alpha)$

   (c) $\mathcal{R}(\alpha^2)$

   (d) None of these

2. $f \in \mathcal{R}(\alpha)$ if

(a) f is continuous on $[a, b]$

(b) f is monotonic on $[a, b]$

(c) f is bounded on $[a, b]$

(d) none of these

3. $\lim\limits_{m \to \alpha} \lim\limits_{n \to \alpha} (\cos(m! \pi x))^{2n} =$

(a) 0

(b) 1

(c) –1

(d) none of these

4. Let $f_n(x) = n^2 x (1 - x^2)^n \quad (0 \le x \le 1,\ n = 1, 2, 3, \ldots)$. Then $\frac{1}{2}$ is the value of

(a) $\lim\limits_{n \to \alpha} f_n(x)$

(b) $\lim\limits_{n \to \alpha} \int_0^1 f_n(x) dx$

(c) $\int_0^1 \left(\lim\limits_{n \to \alpha} f_n(x) \right) dx$

(d) none of these

5. If $\mathcal{A}$ has the property that $f \in \mathcal{A}$ whenever $f_n \in \mathcal{A}$ $(n = 1, 2, 3, \ldots)$ and $f_n \to f$ uniformly on E, then $\mathcal{A}$ is said to be

(a) uniformly closed

(b) pointwise closed

(c) closed

(d) none of these

6. $\int_{-1}^{1}(1-x^2)^n dx$ is

(a) less than $\frac{1}{\sqrt{n}}$ (b) equal to $\frac{1}{\sqrt{n}}$

(c) greater than $\frac{1}{\sqrt{n}}$ (d) none of these

7. Let K be compact and let $f_n \in \mathscr{C}(K)$ $n = 1, 2, 3, ...$ $\{f_n\}$ contains a uniformly convergent subsequence is ___________.

(a) $\{f_n\}$ is pointwise bounded

(b) $\{f_n\}$ is equicontinuous on K

(c) Both (a) and (b) are true

(d) Neither (a) nor (b) is true

8. Suppose the series $\sum_{0}^{\infty} C_n x^n$ converges for $\|x\| < R$ then $\sum_{1}^{\infty} nC_n x^{n-1}$ converges in

(a) $\left(-\frac{1}{R}, \frac{1}{R}\right)$ (b) $(-2R, 2R)$

(c) $(-R, R)$ (d) None of these

9. $\Gamma\left(\frac{1}{2}\right) =$ __________.

(a) π (b) $\sqrt{\pi}$

(c) $\sqrt{\frac{\pi}{2}}$ (d) $\frac{\pi}{2}$

10. The sequence of complex functions $\{\phi_n\}$ is said to be orthonormal if

(a) $\int_a^b \phi_n(x)^2\,dx = 1$ (b) $\int_a^b \phi_n(x)\,dx = 1$

(c) $\int_a^b |\phi_n(x)|^2\,dx = 1$ (d) $\int_a^b \phi_n^2(x)\,dx = 1$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove fundamental theorem of Calculus.

Or

(b) Prove that $f \in \mathcal{R}(\alpha)$ on $[a, b]$ if and only if for every $\varepsilon > 0$ there exists a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$.

12. (a) Prove that the limit of the integral need not be equal to the integral of the limit even if both are finite.

Or

(b) State and prove the Cauchy Criterion for Uniform Convergence.

13. (a) Let α be monotonically increasing on $[a, b]$. Suppose $f_n \in \mathcal{R}(\alpha)$ on $[a, b]$ for $n = 1, 2, 3, \ldots$ and suppose $f_n \to f$ uniformly on $[a, b]$, prove that $f \in \mathcal{R}(\alpha)$ on $[a, b]$ and $\int_a^b f d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$.

Or

(b) If K is a compact metric space, if $f_n \in \mathcal{C}(K)$ for $n = 1, 2, 3, \ldots.$ and if $\{f_n\}$ converges uniformly on K then show that $\{f_n\}$ is equicontinuous on K.

14. (a) Let $\mathcal{B}$ be the uniform closure of an algebra $\mathcal{A}$ of bounded functions. Then show that $\mathcal{B}$ is a uniformly closed algebra.

Or

(b) Suppose ΣC_n converges. Put $f(x) = \sum_{n=0}^{\infty} C_n x^n$ $(-1 < x < 1)$. Then show that $\lim_{x\to 1} f(x) = \sum_{n=0}^{\infty} C_n$.

15. (a) If $f(x)=0$ for all x in some segment J then show that $\lim S_N(\rho:x)=0$ for every $x \in J$.

Or

(b) If $x > 0$ and $y > 0$ then show that

$$\int_0^1 t^{x-1}(1-t)^{y-1}\,dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}.$$

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Suppose $f \in \mathscr{R}(\alpha)$ on $[a, b]$, $m \leq f \leq M$, ϕ is continuous on $[m, M]$ and $h(x)=\phi(f(x))$ on $[a, b]$. Then show that $h \in \mathscr{R}(\alpha)$ on $[a, b]$.

Or

(b) Assume α is increased monotonically and $\alpha' \in \mathscr{R}$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Then prove that $f \in \mathscr{R}(\alpha)$ if and only if $f\alpha' \in \mathscr{R}$ and $\int_a^b f\,d\alpha = \int_a^b f(x)\alpha'(x)\,dx$.

17. (a) Suppose $\{f_n\}$ is a sequence of functions, differentiable on $[a, b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a, b]$. If $\{f_n'\}$ converges uniformly on $[a, b]$, then show that $\{f_n\}$ converges uniformly on $[a, b]$, to a function f, and $f'(x) = \lim_{n\to\infty} f_n'(x)$ $(a \le x \le b)$.

Or

(b) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

18. (a) If γ' is continuous on $[a, b]$ then prove that γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)dt|$.

Or

(b) Let $\{f_n\}$ be a sequence of functions such that $f_n \to f$ uniformly on E in a metric space. Let x be a limit point of E. Then show that $\lim_{t\to x}\lim_{n\to\infty} f_n(t) = \lim_{n\to\infty}\lim_{t\to x} f_n(t)$.

19. (a) State and prove the Stone-Weierstrass theorem.

Or

(b) Given a double sequence $\{a_{ij}\}$ $(i = 1, 2, 3, ...)$, $(j = 1, 2, 3, ...)$, suppose that $\sum_{j=1}^{\infty} |a_{ij}| = b_i$ $(i = 1, 2, 3, ...)$ and Σb_i converges. Then prove that

$$\sum_{i-1}^{\infty}\sum_{j=1}^{\infty} aij = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} aij .$$

20. (a) State and prove Parseval's Theorem.

Or

(b) Define gamma function. Prove that if f is a positive function on $(0, \infty)$ such that (i) $f(x+1) = xf(x)$ (ii) $f(1) = 1$ (iii) $\log f$ is convex then show that $f(x) = \Gamma(x)$.

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ode No. : 5317   **Sub. Code : PHAM 24/ ZMAM 24**

.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2022

Second Semester

Mathematics — Core

DIFFERENTIAL GEOMETRY

(For those who joined in July 2017 onwards)

ne : Three hours   Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

A regular vector valued function of class m is called a

(a) curve of class m

(b) path of class m

(c) differentiable function of class m

(d) function of class m

2. The point P on the curve for which $\overline{r''} = 0$ is called

(a) a singular point

(b) a central point

(c) a point of inflexion

(d) an ordinary point

3. A curve which lies on the tangent surface of C and intersects the generator orthognoally is called

(a) an evolute (b) an involute

(c) a base curve (d) an orthogonal curve

4. The osculating plane at P has ———— contact with the curve at P.

(a) four point

(b) at least four point

(c) three – point

(d) two - point

5. An ordinary point is defined as one for which rank $\begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{pmatrix}$ is

(a) 1 (b) 2

(c) 0 (d) 3

For the paraboloid $x = u,\ y = v,\ z = u^2 - v^2$, E is

(a) $-4uv$ (b) $1+4v^2$

(c) $1+4u^2$ (d) u^2+v^2

The two directions given by $Pdu^2 + 2Q\, dudv + Rdv^2 = 0$ are orthogonal on a surface, if and only if

a) $ER - 2FQ + GP = 0$

b) $ER + 2FQ - GP = 0$

c) $ER - 2QF - GP = 0$

d) $ER - FQ + GP = 0$

A necessary and sufficient condition for a curve $u = u(t),\ v = v(t)$ on a surface $r = r(u,v)$ to be geodesic is that

a) $U\dfrac{\partial T}{\partial \dot{v}} + V\dfrac{\partial T}{\partial \dot{u}} = 0$

b) $U\dfrac{\partial T}{\partial \dot{v}} - V\dfrac{\partial T}{\partial \dot{u}} = 0$

c) $U\dfrac{\partial T}{\partial v} - V\dfrac{\partial T}{\partial u} = 0$

d) $U - V\dfrac{\partial T}{\partial \dot{u}} = 0$

9. The mean curvature μ is defined by

(a) $\mu = K_a K_b$ (b) $\mu = K_a + K_b$

(c) $2\mu = K_a + K_b$ (d) $\frac{1}{2}\mu = K_a + K_b$

10. If ϕ is the angle between the principal normal n to a curve on surface and its surface normal N, then K_n is

(a) $K \sin\phi$ (b) $K \tan\phi$

(c) $K \sec\phi$ (d) $K \cos\phi$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Find the arc length of one complete turn of the circular helix $r(u) = (a\cos u,\ a\sin u,\ bu)$ $-\infty < u < \infty$ where $a > 0$ and obtain the equation of the helix with s as parameter.

Or

(b) Prove that a necessary and sufficient condition for a curve to be a straight line is that $K = 0$ at all points of the curve.

12. (a) Obtain the center C and radius R spherical curvature at a point P on the given curve γ.

Or

(b) Define involute and evolute of a curve and show that the involutes of a circular helix are plane curves.

13. (a) Show that a proper parametric transformation transforms an ordinary point into an ordinary point.

Or

(b) Find E, F, G and H of the anchor ring corresponding to the domain $0 \le u \le 2\pi$, $0 \le v \le 2\pi$,

14. (a) On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes $z =$ constant.

Or

(b) Prove that the curves of the family v^3/u^2 = constant are geodesics on a surface with metric $v^2du^2 - 2uvdudv + 2u^2dv^2$ $(u > 0, v > 0)$.

15. (a) Define the geodesic curvature prove that the geo desic curvature vector of any curve is orthogonal to the curve.

Or

(b) With usual notations, prove that $K_n = \dfrac{Ldu^2 + 2Mdudv + NdN^2}{Edu^2 + 2Fdudv + GdN^2}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Calculate the torsion and curvature of the cubic curve $r = (u, u^2, u^3)$.

Or

(b) State and prove Serret – Frenet formula.

17. (a) If $r = r(s)$ is the given curve γ, prove that the center C and radius R of spherical curvature at a point P on γ are given by $C = r + \rho n + \sigma p' b$, $R = \sqrt{\rho^2 + \sigma^2 p'^2}$.

Or

(b) Show that a necessary and sufficient condition for a curve to be helix is that the ratio of the curvature to torsion is constant at all points.

18. (a) (i) Show that the metric is invariant under a parametric transformation.

(ii) Find angle between parametric curves.

Or

(b) If (l', m') are the direction coefficients of a line which makes an angle $\pi/2$ with the line whose direction coefficients are (l, m), then prove that $l' = -\dfrac{1}{H}(Fl + Gm)$, $m' = \dfrac{1}{H}(El + FM)$.

19. (a) (i) Prove that the two directions given by $Pdu^2 + 2Q\,dudv + Rdv^2 = 0$ are orthogonal on a surface if and only if $ER - 2QF + GP = 0$.

(ii) Also prove that if θ is the angle between the two curves, then $\tan\theta = \dfrac{2H(Q^2 - PQ)^{1/2}}{ER - 2FQ + GP}$.

Or

(b) Prove that any curve u=u(t), $v = v(t)$ on a surface $r = r(u, v)$ is a geodesic if and only if the principal normal at every point on the curve is normal to the surface.

20. (a) State and prove Liouville's formula.

Or

(b) If K is the normal curvature in a direction making an angle ψ with the principal direction γ = constant, then prove that $K = K_a \cos^2\psi + K_b \sin^2\psi$ where K_a and K_b are principal curvatures at the point P on the surface.

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ges) **Reg. No. :** ....

ue No. : 5315 **Sub. Code : PMAM 22/ ZMAM 22**

:. (CBCS) DEGREE EXAMINATION, APRIL 2022

Second Semester

Mathematics — Core

ANALYSIS — II

(For those who joined in July 2017 onwards)

: Three hours Maximum : 75 marks

PART A — $(10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

Let $P = \{0, 0.2, 0.8, 0.9, 1\}$ be a partition of $[0,1]$ which one of the following is refinement of P

(a) $\{0, 0.2, 0.7, 0.9, 1\}$

(b) $\{0, 0.2, 0.6, 0.8, 0.93, 1\}$

(c) $\{0, 0.2, 0.3, 0.4, 0.8, 0.9, 1\}$

(d) $\{0, 0.2, 0.6, 0.7, 0.8, 1\}$

2. $f \in R(\alpha)$ on $[a,b]$ if and only if for every $\varepsilon > 0$

(a) $U(P,f,\alpha) - L(P,f,\alpha) < \varepsilon$ for every partition P

(b) there exists a partition P such that $U(P,f,\alpha) - L(P,f,\alpha) < \varepsilon$

(c) there exists a partition P such that $L(P,f,\alpha) - U(P,f,\alpha) < \varepsilon$

(d) $L(P,f,\alpha) - U(P,f,\alpha) < \varepsilon$ for every partition P

3. Let $s_{m,n} = \dfrac{m}{m+n}$; $\begin{matrix} m = 1,2,3,.. \\ n = 1,2,3,.. \end{matrix}$, then $\lim\limits_{m\to\infty} \lim\limits_{n\to\infty} s_{m,n}$ is

(a) 1 (b) $\frac{1}{2}$

(c) ∞ (d) 0

4. A sequence $\{f_n\}$ converges to f w.r.t. the metric of $\mathscr{C}(X)$ if and only if

(a) $f_n \to f$ on X

(b) f is continuous on X

(c) $f_n \to f$ uniformly on X

(d) f_n and f are continuous on X

If K is compact, if $f_n \in \mathscr{C}(K)$ for $n = 1, 2, 3.,,,$ and if $\{f_n\}$ is ———— and ———— on K then $\{f_n\}$ is uniformly bounded on K.

(a) pointwise bounded and continuous

(b) pointwise bounded and equicontinuous

(c) continuous and differentiable

(d) a sequence of continuous and bounded functions

Let $f_n(x) = \dfrac{x^2}{x^2 + (1-nx)^2}$ $(0 \le x \le 1,\ n = 1,2,3,...)$ then $f_8(1/8)$ is

(a) 0 (b) 1

(c) ∞ (d) 8

The set of continuous functions on $[a,b]$ is the uniform closure of the set of polynomials on $[a,b]$. This statement is known as

(a) Lagrange theorem

(b) Weierstrass theorem

(c) Stone's theorem

(d) Cauchy's theorem

8. $\lim\limits_{n\to\infty}\left(1+\dfrac{x}{n}\right)^n = ?$

(a) e^x (b) e^n

(c) e (d) 1

9. The Dirichlet Kernel $D_N(x)$ is also equal to

(a) $\dfrac{\sin(N+1)x}{\sin x}$ (b) $\dfrac{\sin(N+1/2)x}{\sin(x/2)}$

(c) $\dfrac{\sin(N+1/2)x}{\sin x}$ (d) $\dfrac{\sin(N+1)x}{\sin(x/2)}$

10. The value of $\Gamma(1/2)$ is

(a) 1/2 (b) 1

(c) π (d) $\sqrt{\pi}$

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If P^* is a refinement of P, prove that $L(P, f, \alpha) \le L(P^*, f, \alpha)$.

Or

(b) If f is monotonic on $[a, b]$ and if α is continuous on $[a, b]$, prove that $f \in R(\alpha)$.

12. (a) Give an example to show that a convergent series of continuous functions may have a discontinuous functions sum.

Or

(b) State and prove the Weierstrass test for uniform convergence.

13. (a) Let α be monotonically increasing on $[a, b]$ suppose $f_n \in \mathscr{R}(\alpha)$ on $[a, b]$ for $n = 1, 2, 3, \ldots$ and suppose $f_n \to f$ uniformly on $[a, b]$. Prove that $f_n \in \mathscr{R}(\alpha)$ on $[a, b]$ and $\int_a^b f\, d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$.

Or

(b) If K is compact, if $f_n \in \mathscr{C}(K)$ for $n = 1, 2, 3 \ldots$ and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, then prove that $\{f_n\}$ is uniformly bounded on K.

14. (a) Define an algebra and the uniform closure of an algebra. Let $\mathscr{B}$ be the uniform closure of an algebra $\mathscr{A}$ of bounded functions. Prove that $\mathscr{B}$ is a uniformly closed algebra.

Or

(b) Suppose Σc_n converges. Put $f(x) = \sum_{n=0}^{\infty} c_n x^n (-1 < x < 1)$ prove that $\lim_{x\to 1} f(x) = \sum_{n=0}^{\infty} c_n$.

15. (a) Define the Dirichlet Kernel $D_n(x)$ and show that $D_N(x) = \dfrac{\sin(N + 1/2)^x}{\sin(x/2)}$ and $s_N(f; x) = \dfrac{1}{2\pi} \int_{-\pi}^{\pi} f(x - t) D_N(t)\, dt$.

Or

(b) If $x > 0$ and $y > 0$, prove that $\int_0^1 t^{x-1}(1-t)^{y-1} dt = \dfrac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If $f_1, f_2 \in \mathscr{R}(\alpha)$ on $[a,b]$, prove that $f_1 + f_2$, $cf_1 \in \mathscr{R}(\alpha)$ for every constant c and $\int_a^b (f_1 + f_2) d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$, $\int_a^b cf_1 d\alpha = c \int_a^b f_1 d\alpha$.

Or

(b) Assume α increases monotonically and $\alpha' \in \mathscr{R}$ on $[a,b]$. Let f be a bounded real function on $[a,b]$. Prove that $f \in \mathscr{R}(\alpha)$ if and only if $f\alpha' \in \mathscr{R}$ and show that $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)\, dx$.

17. (a) If γ' is continuous on $[a, b]$, prove that γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma'(t)|\, dt$.

Or

(b) Suppose $f_n \to f$ uniformly on a set E in a metric space. Prove that $\lim_{t\to x} \lim_{n\to\infty} f_n(t) = \lim_{n\to\alpha} \lim_{t\to x} f_n(t)$.

18. (a) Prove that the exists a real continuous function on the real line which is nowhere differentaiable.

Or

(b) If $\{f_n\}$ is a pointwise bounded sequence of complex functions on a countable set E prove that $\{f_n\}$ has a subsequence $\{f_{n_k}\}$ such that $\{f_{n_k}(x)\}$ converges for every $x \in F$.

19. (a) If f is a continuous complex function on $[a, b]$, prove that there exists a sequence of polynomials p_n such that $\lim_{n\to\alpha} P_n(x) = f(x)$.

Or

(b) State and prove Taylor's theorem.

20. (a) Suppose $a_0, a_1, \ldots, a_n$ are complex numbers, $n \geq 1$, $a_n \neq 0$. $P(z) = \sum_0^n a_k z^k$. Prove that $P(z) = 0$ for some complex numberz.

Or

(b) If f is a positive function on $(, \infty)$ such that (i) $f(x+1) = xf(x)$ (ii) $f(1) = 1$ (iii) $\log f$ is convex, prove that $f(x) = \Gamma(x)$.

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:zes) **Reg. No. :** ....

ɔae No. : 5330 **Sub. Code : PMAM 41**

.. (CBCS) DEGREE EXAMINATION, APRIL 2022

Fourth Semester

Mathematics — Core

FUNCTIONAL ANALYSIS

For those who joined in July 2017 onwards)

Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

hoose the correct answer :

the complex number C are normed linear space
.en

) $\|x\| = \|x\|$ (b) $\|x\| = |x|$

) $\|x\| \le |x|$ (d) $\|x\| \ge |x|$

2. A normed linear space has one of the following property

(a) $\|\alpha x\| = |\alpha|\|x\|$ (b) $\|\alpha x\| = \alpha x$

(c) $\|\alpha x\| \le |\alpha|\|x\|$ (d) $\|\alpha x\| \ge |\alpha|\|x\|$

3. The conjugate space of a normed linear space is

(a) linear space

(b) normed linear space

(c) banach space

(d) none of these

4. A banach space B is reflexive iff

(a) B^* is not relfexive

(b) B^* is symmetric

(c) B^* is relfexive

(d) B^* is transitive

5. A mapping $T \to T^*$ then $(\alpha T_1 + \beta T_2)^*$ is

(a) I (b) $\alpha T_1^* + \beta T_2^*$

(c) $\alpha T_1 + \beta T_2$ (d) $\alpha T_2^* + \beta T_1^*$

x and y are orthogonal in an inner product
ace, then

$(x,y) = \|x\|\,\|y\|$ (b) $(x,y) = 0$

$(x,y) = 1$ (d) $(x,y) \le \|x\|$

. orthonormal set in a Hilbert space H consists

orthogonal vectors

mutually orthogonal unit vectors

orthogonal unit vectors

none of these

ɜ Bessel's inequality is

$\Sigma|x_i e_i|^2 \ge \|x\|^2$ (b) $\Sigma|x_i e_i|^2 \le \|x\|^2$

$\Sigma|x_i e_i|^2 > \|x\|^2$ (d) $\Sigma|x_i e_i|^2 < \|x\|^2$

. operator T on H is unitary then

$TT^* = I$ (b) $TT^* = T^*T = I$

$TT^* \ne T^*T$ (d) None of these

10. If $\det([\alpha_{ij}]) \ne 0$ iff

(a) $[\alpha_{ij}]$ is singular

(b) $[\alpha_{ij}]$ is non singular

(c) $[\alpha_{ij}]$ identity matrix

(d) none of these

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove Minkowski's unequality.

Or

(b) Prove that if M is a closed linear subsapce of a normed liner space N and x_0 is a vector not in M, then there exists a functional f_0 in N^* such that $f_0(M) = 0$ and $f_0(x_0) \ne 0$.

12. (a) State and prove closed graph theorem.

Or

(b) If P is a projection on a Banach space B, and if M and N are its range and null space, then M and N are closed linear subspaces of B such that $B = M \oplus N$ – prove.

13. (a) If B is a complex banach space whose norm obeys the parallelogram law, and if an inner product is defined on B by $4(x, y) = \|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2$ then prove that B is a Hilbert space.

Or

(b) If M is a closed linear subspace of a Hilbert space H then $H = M \oplus M^{\perp}$ – prove.

14. (a) Let $\{e_1, e_2, .., e_n\}$ be a finite orthonormal set in a Hilbert space H. If x is any vector in H, then $\sum_{i=1}^{n} |(x_i e_i)|^2 \leq \|x\|^2$ further $x - \sum_{i=1}^{n} (x_i e_i) e_i \perp e_j$ for each j. – Prove.

Or

(b) Prove that if A_1 and A_2 are self-adjoint operators on H, then their product $A_1 A_2$ is self-adjoint iff $A_1 A_2 = A_2 A_1$.

15. (a) If P is the projection on a closed linear subspace M of H then M is invariant under an operator $T \Leftrightarrow TP = PTP$ - Prove.

Or

(b) It T is normal, then the M_i's are pairwise orthogonal.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If N and N' are normed linear spaces, then the set $B(N, N')$ of all continuous linear transformations of N and N' is itself a normed linear space with respect to point wise linear operations and the norm defined by $\|T\| = \sup\{\|T(x)\| : \|x\| \leq 1\}$ further, if N' is a Banach space, then $B(N, N')$ is also a Banach space – Prove.

Or

(b) Let M be a liner subspace of a normed linear space N and let f be a functional defined on M. If x_0 is a vector not in M, and if $M_0 = M + [x_0]$ is the linear subspace spanned by M and x_0, then prove that f can be extended to a functional f_0 defined on M_0 such that $\|f_0\| = \|f\|$.

17. (a) State and prove open mapping theorem.

Or

(b) Prove that if N is a normed linear space, then the closed unit sphere in S^* in N^* is a compact Hausdroff space in the weak topology.

18. (a) State and prove uniform boundedness theorem.

Or

(b) A closed convex subset C of a Hilbert space if contains a unique vector of smallest norm – prove.

19. (a) Let H be a Hilbert space and let f be an arbitrary functional in H^*. Then prove that there exists a unique vector y in H such that $f(x) = (x, y)$ for every $x \in H$.

Or

(b) If A is a positive operator on H, then prove that If A is an singular in particular, $I + T^*T$ and $I + TT^*$ are non-singular for an arbitrary operator T on H.

20. (a) If N_1 and N_2 are normal operators on H with the property that either commutes with the adjoint of the other, then prove that $N_1 + N_2$ and $N_1 N_2$ are normal.

Or

(b) If P is a projection on H with range M and null space N then $M \perp N \Leftrightarrow P$ is self-adjoint and $N = M^{\perp}$ - Prove.

(7 pages)

Reg. No. : ....

Code No.: 6022 **Sub. Code: PMAM 41**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(For those who joined in July 2017-2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The normed linear space N is a ________ space with respect to the metric d defined by $d(x, y) = \|x - y\|$
   (a) Metric (b) Complete
   (c) Hilbert (d) Inner

2. The spaces R and C the real numbers and the complex numbers are the simplest of all ________ spaces
   (a) Complex (b) Hilbert
   (c) Normed linear (d) None

3. The conjugate' space of N* is called as ________ conjugate.
   (a) second (b) dual of N
   (c) third (d) first

4. The isometric isomorphism $x \to F_x$ is called the ______ of N into N**.
   (a) banach (b) natural imbedding
   (c) surjective (d) injuctive

5. A ________ space is a complex banach space whose norm arises from the inner product.
   (a) Hilbert (b) Banach
   (c) Inner product (d) Banach algebra

6. A ________ set in a Hilbert space H is a non empty subset of H which consists of mutually orthogonal unit vectors.
   (a) Hilbert (b) Empty
   (c) Orthonormal (d) Banach

7. The value of T^{**} = ________.
   (a) T (b) T^*
   (c) T^{1} (d) T_1

8. The operator T is self adjoint if A = ________.
   (a) A (b) A^2
   (c) A^* (d) A^{**}

9. The value of det(1) = ________.
   (a) 0 (b) 1
   (c) –1 (d) 2

10. $\det(T) \neq 0$ if and only if T is ________.
    (a) singular (b) unitary
    (c) non singular (d) self adjoint

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that if M is a closed linear subspace of a normed linear space N and x_0 is a vector not in M, then there is a functional f_0 in N^* such that $f_0(M) = 0$ and $f_0(x_0) \neq 0$.

    Or

    (b) Let N and N' be normed linear spaces and T a linear transformation of N into N'. Then prove that the following are equivalent.
    (i) T is continuous
    (ii) T is continuous at the origin
    (iii) There exists a real number $K \geq 0$ with the property that $\|T(x)\| \leq K\|x\|$ for every x in N.
    (iv) If $S\{x : \|x\| \leq 1\}$ is the closed unit sphere in N, then its image $T(S)$ is a bounded set in N'.

12. (a) Prove that if n is a normed linear space then the closed unit sphere S^* in N^* is a compact Hausdorff space in the weak* topology.

    Or

    (b) Prove that if B and B' are Banach spaces and if T is a linear transformation of B into B' then T is continuous if and only if its graph is closed.

13. (a) Prove that if M is a proper closed linear subspace of a Hilbert space H, then there exists a non zero vector z_0 in H such that $z_0 \perp M$.

    Or

    (b) Prove that if x and y are any two vectors in a Hilbert space H, then $|\langle x, y\rangle| \leq \|x\|\|y\|$.

14. (a) The adjoint operation $T \to T^*$ on B(H) has the following properties Prove them

(i) $(T_1 + T_2)^* = T_1^* + T_2^*$

(ii) $(\alpha T)^* = \bar{\alpha} T^*$

(iii) $(T_1 T_2)^* = T_2^* T_1^* T^{**} = T$

Or

(b) Prove that if T is an operator on H, for which $(Tx, x) = 0$ for all x then $T = 0$.

15. (a) Prove that an operator T on H is unitary if and only if it is an isometric isomorphism of H onto itself.

Or

(b) If T is normal, then prove that the Mi's are pairwise orthogonal.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let M be a closed linear space of a normed linear space N. If the norm of a coset $x+M$ in the quotient space N/M is defined by $\|x + M\| = \inf\{\|x + m\| : m \in M\}$ then prove that N/M is a normed linear space.

Or

(b) State and prove Hahn banach theorem.

17. (a) Prove that if B and B' are Banach spaces, and if T is a continuous linear transformation of B onto B', then T is an open mapping.

Or

(b) If T is an operator on a nls N, then prove that its conjugate T^* defined by $[T^*(f)](x) = f(T(x))$ is an operator on N^* and the mapping $T \to T^*$ is an isometric isomorphism of $\mathfrak{B}(N^*)$ into $\mathfrak{B}(N^*)$ which reverses products and preserves the identity transformation.

18. (a) Prove that if M and N are closed linear subspaces of a Hubert space H such that $M \perp N$, then the linear subspace $M + N$ is also closed.

Or

(b) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

19. (a) Let H be a Hilbert space, and let f be an arbitrary functional in H^*, then prove that there exists a unique vector y in H such that $f(x) = (x, y)$ for every x in H.

Or

(b) Prove that if $\{e_i\}$ is an ortho normal set in a Hilbert space H, and if x is an arbitrary vector in H then $x - \sum (x, e_i) e_i \perp e_j$ for each j.

20. (a) Prove that if $B = \{e_i\}$ is a basis for H, then the mapping $T \to [T]$, which assigns to each operator T its matrix relative to B, is an isomorphism of the algebra $\mathfrak{B}(H)$ onto the total matrix algebra A_n.

Or

(b) (i) Prove that $\|N^2\| = \|N\|^2$ if N is normal operator on H.

(ii) Also prove that if T is an operator on H, then T is normal iff its real and imaginary parts commute.

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7 Pages)

Reg. No. : ....

'ode No. : 5333 **Sub. Code : PMAM 44**

1.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2022

Fourth Semester

Mathematics — Core

TOPOLOGY — II

(For those who joined in July 2017 onwards)

me : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

A space having a countable dense subset is called a

(a) Lindelof space (b) Separable space
(c) Hausdorff space (d) Regular space

Which one of the following is not true

(a) A regular space is Hausdorff
(b) A normal space is regular
(c) A hausdorff space is regular
(d) A product of regular spaces is regular

3. Let X be a well-ordered set. Then every interval of the form $(x, y]$ is

(a) closed in X

(b) open in X

(c) neither open nor closed in X

(d) both open and closed in X

4. Consider the two statements

A : A normal space is completely regular

B : A completely regular space in regular. Then

(a) Both A and B are true

(b) Neither A nor B is true

(c) A is true but B is not true

(d) A is not true but B is true

5. Every ———— space X with a countable basis is metrizable.

(a) Hausdorff (b) Normal

(c) Regular (d) Topological

If we divide the internal $[-r, r]$ into three equal intervals of length $\frac{2}{3}r$, then the middle interval is

(a) $\left[-r, \frac{-2}{3}r\right]$ (b) $\left[\frac{2}{3}r, r\right]$

:c) $\left[-\frac{r}{3}, \frac{r}{3}\right]$ (d) $\left[\frac{1}{3}r, r\right]$

3iven a set A that is strictly partially ordered, in vhich every simple ordered subset has an upper ound, A itself has a maximal element ———— his result is known as.

a) Urysohn lemma

b) Zorn's lemma

:) Tube lemma

1) The sequence lemma

. collection B of subsets of X is said to be ountable locally finite it B can be written as

) the countable union of collections B_n each of which is locally finite

) the countable union of open sets

) the countable union of no where dense subsets

) the countable union of collections B_n each of which is locally connected

9. The interior of Q as a subset of $\mathbb{R}$ is

(a) ϕ (b) Q

(c) $R-Q$ (d) $\mathbb{R}$

10. The interior of $Q \times R$ as a subset of $\mathbb{R}_2$

(a) $R \times R$ (b) $Q \times R$

(c) $Q \times Q$ (d) ϕ

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Suppose that X has a countable basis. Prove that every open covering of X contains a countable sub collection covering X.

Or

(b) Prove that a subspace of a regular space is regular.

12. (a) Show that every compact Hausdorff space is normal.

Or

(b) Define a completely regular space. And Show that a normal space is completely regular and a completely regular space is regular.

13. (a) Let X be a compact Hausdorff space. Show that X is metrizable if and only if X has a countable basis.

Or

(b) Show that the Tietze extension theorem implies the Urysohn lemma.

14. (a) Let A be a locally finite collection of subsets of X. Prove that the collection B of the closures of the elements of A is locally finite.

Or

(b) Show that if X has a countable basis, a collection A of subsets of X is countably locally finite if and only if it is countable.

15. (a) Define a Baire space. Give an example of a topological space which is not a Baire space (with justification).

Or

(b) Prove that any open. Subspace Y of a Baire space X is itself a Baire space.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Prove that a subspace of a second countable space is second countable but a subspace of Lindelot space need not be Lindelot.

Or

(b) Let X be a topological space. Let one-point sets in X be closed. Prove that X is regular if and only if given a point x of X and a neighborhood U of x, there is a neighborhood V of x such that $\overline{V} \subseteq U$.

17. (a) Prove that every regular space with a countable basis is normal.

Or

(b) State and prove the Urysohn lemma.

18. (a) Prove that every regular space X with a countable basis is metrizable.

Or

(b) State and prove Tietze extension theorem.

19. (a) State and prove Tychonoff theorem.

Or

(b) Let X be a metrizable space. If A is an open covering of X, prove that there is an open covering ε of X refining of that is countably locally finite.

20. (a) If X is a compact Hausdorff space of a complete metric space, prove that X is a Baire space.

Or

(b) Let X be a space; let (Y,d) be a metric space. Let $f_n : X \to Y$ be a sequence of continuous functions such that $f_n(x) \to f(x)$ for all $x \in X$ where $f : X \to Y$. If X is a Baire space, prove that the set of points at which f is continuous is dense in X.

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Reg. No. : ....

de No. : 5675 **Sub. Code : ZMAE 21**

(CBCS) DEGREE EXAMINATION, APRIL 2022

Second Semester

Mathematics

Elective — CLASSICAL MECHANICS

For those who joined in July 2021 onwards)

Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

hoose the correct answer :

ngular momentum of a particle ————

a) $\overline{L} = \overline{r} . \overline{P}$ (b) $\overline{L} = \overline{r} \times \overline{P}$

c) $\overline{r} . \overline{P} = 0$ (d) $\overline{r} \times \overline{P} = \overline{0}$

f the total external force is zero, the total linear momentum is ————

a) Conserved (b) Zero

c) Negative (d) Positive

3. Principle of virtual work ————

(a) $\sum_i F_i^{(a)} \times \delta r_i = 0$ (b) $\sum_i F_i^{(a)} . \delta r_i = a$

(c) $\sum_i F_i^{(a)} . \delta r_i - 0$ (d) $\sum_i F_i^{(a)} \times \delta r_i = a$

4. Canonical momentum is ————

(a) $P_j = \dfrac{\partial T}{\partial q_j}$ (b) $P_j = \dfrac{\partial L}{\partial q_j}$

(c) $P_j = \dfrac{\partial U}{\partial q_j}$ (d) $P_j = \dfrac{\partial P}{\partial q_j}$

5. The generalized momentum conjugate to a cyclic co-ordinate is ————

(a) zero (b) negative

(c) positive (d) conserved

6. If system is spherically symmetric, the components of ———— are conserved.

(a) Linear momentum

(b) Energy

(c) Angular momentum

(d) Kinetic energy

If q_j is cyclic, then $\dfrac{\partial L}{\partial q_j} =$ ————

(a) zero (b) q_j

(c) $-q_j$ (d) conserved

For monogenic system, hamilton's principle $I =$ ————

(a) $\int_{t_1}^{t_2} (T+V)\,dt$ (b) $\int_{t_1}^{t_2} (TV)\,dt$

(c) 0 (d) $\int_{t_1}^{t_2} (T-V)\,dt$

The position E_0 of the earth in its actual orbit around the sun when its nearest to sun is called ————

(a) uphelian (b) helicoid

(c) center of orbit (d) perihelion

Anne drawn from planet to sun sweeps equal areas in equal amounts of time. This is ————

(a) Kepler's second law

(b) Kepler's third law

(c) Kepler's first law

(d) Law of time constrain

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove conservation theorem for the angular momentum for a particle.

Or

(b) Explain the types of constraints.

12. (a) State and prove D'Alembert's principle.

Or

(b) Explain Atwood's machine.

13. (a) Explain about Hamilton's principle.

Or

(b) Derive shortest difference between two points in a plane is straight line by applying Lagrange's equation.

14. (a) Explain about reduction to equivalent one body problem.

Or

(b) Prove that the central force of motion is always motion in a plane.

15. (a) Discuss about the motion in time in Kepler's problem.

Or

(b) Discuss about the Kepler problem : inverse square law of force.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove conservation theorem for the linear momentum of a system of particles.

Or

(b) State and prove conservation theorem for the angular momentum of a system of particles.

17. (a) Derive Lagrangean equation for holonomic constraint.

Or

(b) Explain about applications of Lagrangian formulation.

18. (a) Derive lagrange's equation from the Hamilton's principle.

Or

(b) Discuss about some techniques of calculus of variation.

19. (a) State and prove virial theorem.

Or

(b) Apply virial theorem, derive Boyle's law for perfect gases.

20. (a) Write a note on Laplace Runge Lenze vector.

Or

(b) Derive the differential equation for the orbit.

———————

(8 pages) **Reg. No. :** ....

Code No. : 6374 **Sub. Code : ZMAE 21**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Second Semester

Mathematics

Elective — CLASSICAL MECHANICS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The equation of motion is a differential equation of order

(a) 3 (b) 0

(c) 1 (d) 2

2. If there exist holonomic constrains expressed in 5 equations, then the system of 50 particles has ———— degrees of freedom.

(a) 55 (b) 45

(c) 145 (d) 150

3. The Lagrangian L is defined as

(a) $T+V$

(b) $T-V$

(c) $V-T$

(d) $T.V$

4. The equation $\sum_i F_i^{(a)} \cdot \delta r_i = 0$ is called

(a) the principle of virtual work

(b) the equation of motion

(c) the Lagrangian equation

(d) the Maxwell equation

5. The total length of any curve going between points 1 and 2 is

(a) $\int_{x_1}^{x_2}\left(1+\left(\frac{dy}{dx}\right)^2\right)dx$

(b) $\int_{x_1}^{x_2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$

(c) $\int_{x_1}^{x_2}\left(1+\left(\frac{dy}{dx}\right)^{1/2}\right)dx$

(d) $\int_{x_1}^{x_2}\sqrt{\left(\frac{dy}{dx}\right)+\left(\frac{dy}{dx}\right)^2}\,dx$

6. For any parametric family of curves $J(\alpha)=\int_{x_1}^{x_2} f\left(y(x,\alpha),\dot{y}(x,\alpha),x\right)dx$ the condition for obtaining a stationary point is

(a) $\left.\left(\frac{dJ}{dx}\right)_{\alpha=0}\right| = 0$ (b) $\left.\left(\frac{dJ}{d\alpha}\right)_{\alpha=0}\right| = 0$

(c) $\left.\left(\frac{dJ}{d\alpha}\right)_{x=0}\right| = 0$ (d) $\left.\left(\frac{dJ}{d\alpha}\right)_{y=0}\right| = 0$

7. The areal velocity is

(a) $n^2\dot{\theta}$ (b) $\frac{1}{2}r^2\theta$

(c) $\frac{1}{2}r^2\dot{\theta}$ (d) $r^2\dot{\theta}$

8. Consider a plot of V' against r for the specific case of an attractive inverse square law of force $f=-\frac{k}{r^2}$. The potential energy for this force is

(a) $V=\frac{k}{r}$ (b) $V=\frac{k}{r^2}$

(c) $V=-\frac{k}{r}$ (d) $V=0$

9. The eccentric anomaly ψ is defined by the relation

(a) $r=a(1+e\cos\psi)$ (b) $r=a(1-e\cos\psi)$

(c) $r=a(1-\cos\psi)$ (d) $\psi=\frac{ae}{r^2}$

10. The Kepler's equation is

(a) $\omega t=\psi-e\sin\psi$ (b) $\omega t=\psi+e\sin\psi$

(c) $\omega t=\psi-e\cos\psi$ (d) $\omega t=\psi+e\cos\psi$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove the conservation theorem for the angular momentum of a particle.

Or

(b) Define a rigid body and show that in a rigid body the internal forces do no work. What can you say about the internal potential of such body.

12. (a) Obtain the Lagrange equations of motion for a spherical pendulum.

Or

(b) Derive the Lagrange equation of motion of a bead sliding on a uniformly rotating wire in a force-free space.

13. (a) Show that the shortest distance between two points in a plane is a straight line.

Or

(b) Explain Braschistochrone problem.

14. (a) Show that the central force motion of two bodies about their center of mass can always be reduced to an equivalent one-body problem.

Or

(b) Two particles move about each other in circular orbits under the influence of gravitational forces, with a period τ. Their motion is suddenly stopped at a given instant of time, and they are then released and allowed to fall into each other. Prove that they collide after a time $\tau/4\sqrt{2}$

15. (a) Obtain the differential equation for the orbit if the force law f is known.

Or

(b) Prove that for the Kepler problem there exists a conserved vector A defined by $\overline{A} = \overline{P} \times \overline{L} - m\,k\frac{\overline{r}}{r}$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions choosing either (a) or (b).

16. (a) State and prove the conservation theorem for the linear momentum of a system of particles.

Or

(b) (i) Explain holonomic and nonholonomic constraints with suitable examples.

(ii) State the two types of difficulties due to constraints in the solution of mechanical problems.

17. (a) Derive Lagrange's equation of motion from D'Alembert's principle.

Or

(b) Show that the kinetic energy of a system can always be written as the sum of three homogeneous functions of the generalized velocities.

18. (a) Derive the Euler-Lagrange differential equations.

Or

(b) Derive Lagrange's equations for non holonomic systems.

19. (a) State and prove Kepler's second law of planetary motion.

Or

(b) A particle moves in a central force field given by the potential $V = -k\frac{e^{-ar}}{r}$ where k and a are positive constant. Using the method of the equivalent one-dimensional potential discuss the nature of the motion.

20. (a) Obtain the equation of motion for the particle moving under the influence of a central force $f = -k/r^2$.

Or

(b) (i) For the Kepler's equation $\omega t = \psi - e\sin\psi$ prove that $\tan\theta/2 = \sqrt{\frac{1+e}{1-e}}\tan\psi/2$

(ii) Derive the orbit equation for the Kepler problem using Laplace–Runge–Lenz vector.

(7 pages)

Reg. No. : ....

Code No. : 6381 **Sub. Code : ZMAE 31**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Third Semester

Mathematics

Elective — ALGEBRAIC NUMBER THEORY

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Which of the following Diophantine equation cannot be solved?
   (a) $6x+51y=22$ (b) $33x+14y=115$
   (c) $14x-35y=93$ (d) $11x+13y=21$

2. The linear Diophantine equation $ax+by=c$ has a solution if and only if ————
   (a) $\gcd(a,c)|b$ (b) $\gcd(a,b)|c$
   (c) $\gcd(c,b)|a$ (d) $c|\gcd(a,b)$

3. A linear combination of integers a and b is
   (a) ab
   (b) $\frac{a}{x}+\frac{b}{y}$, x and y are integers
   (c) $ab=1$
   (d) $ax+by$, x and y are integers

4. Let a and b be integers, not both zero. Then a and b are relatively prime iff there exists integers x and y such that
   (a) $1=ax+by$ (b) $2=ax+by$
   (c) $ab=ax+by$ (d) $a-b=ax+by$

5. The number of prime is
   (a) finite (b) infinite
   (c) uncountable (d) 1729

6. Two integers a and b, not both of which are zero, are said to be relatively prime if
   (a) $\gcd(a,b)=a$ (b) $a|b$
   (c) $\gcd(a,b)=1$ (d) $b|a$

7. If a is a solution of $P(x) \equiv 0 (\bmod n)$ and $a \equiv b (\bmod n)$, then

(a) ab is also a solution

(b) $a+b$ is also a solution

(c) $a-b$ is also a solution

(d) b is also a solution

8. In n is an odd pseudo prime, then $2^n - 1$ is

(a) pseudo prime (b) prime

(c) irrational (d) not pseudo prime

9. If m and n are relatively prime integers then $\varphi(mn) =$ ————

(a) $\varphi(m) + \varphi(n)$ (b) $\varphi(m) / \varphi(n)$

(c) $\varphi(m) - \varphi(n)$ (d) $\varphi(m)\varphi(n)$

10. If p is a prime and a is any integer then $a^p - a$ is

(a) a multiple of p^2 (b) a multiple of $p-1$

(c) a multiple of $2p$ (d) a multiple of p

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Find all solutions in positive integer $5x + 3y = 52$.

Or

(b) If u and v are relatively prime positive integers whose product uv is a perfect square, then prove that u and v are both perfect squares.

12. (a) Prove that the equation $x + 2y^3 + 4z^3 = 9w^3$ has no nontrivial solution.

Or

(b) Determine the solution of the Diophantine equation $x^2 + 3y^2 + 5z^2 + 7xy + 9yz + 11zx = 0$.

13. (a) For any positive real number x, prove that $\langle a_0, a_1, \ldots, a_{n-1}, x \rangle = \dfrac{xh_{n-1} + h_{n-2}}{xk_{n-1} + k_{n-2}}$.

Or

(b) Let $\theta = \langle a_0, a_1, a_2, \ldots \rangle$ be a simple continued fraction. Then $a_0 = [\theta]$. Further more if θ_1 denotes $\langle a_0, a_1, a_2, \ldots \rangle$ then prove that $\theta = a_0 + 1/\theta_1$.

14. (a) Let ξ denote any irrational number. If there is a rational number $\frac{a}{b}$ with $b \geq 1$ such that $\left|\xi - \frac{a}{b}\right| < \frac{1}{2b^2}$ then prove that $\frac{a}{b}$ equals one of the convergents of the simple continued fraction expansion of ξ.

Or

(b) If an irreducible polynomial $p(x)$ divided a product $f(x)g(x)$, then prove that $p(x)$ divides at least one of the polynomials $f(x)$ and $g(x)$.

15. (a) Let m be a negative square-free rational integer Prove : the field $\mathbb{Q}(\sqrt{m})$ has units ± 1, and these are the only units except in the cases $m = -1$ and $m = -3$. The units for $\mathbb{Q}(i)$ are ± 1 and $\pm i$. The units for $\mathbb{Q}\sqrt{-3}$ are ± 1, $(1 \pm \sqrt{-3})/2$ and $\left(-1 \pm \sqrt{-3}/2\right)$

Or

(b) Prove that the integers of any algebraic number field form a ring.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)
Each answer should not exceed 600 words.

16. (a) Find all solutions in integers of the simultaneous equations $20x + 44y + 50z = 10$, $17x + 13y + 11z = 19$.

Or

(b) Find all integers x and y such that $147x + 258y = 369$.

17. (a) Determine whether the equation $3x^2 + 5y^2 + 7z^2 + 9xy + 11yz + 13zx = 0$.

Or

(b) Prove that the equation $y^2 = x^3 + 7$ has no solution in integers.

18. (a) Prove that the equations $h_i k_{i-1} - h_{i-1} k_i = (-1)^{i-1}$ and $r_i - r_{i-1} = \frac{(-1)^{i-1}}{k_i k_{i-1}}$ hold for $i \geq 1$.

Or

(b) Prove that the value of any infinite simple continued fraction $\langle a_0, a_1, a_2, \ldots \rangle$ is irrational.

19. (a) If a/b is a rational number with positive denominator such that $\left|\xi - \frac{a}{b}\right| < \left|\xi - \frac{h_n}{k_n}\right|$ for some $n \geq 1$, then $b > k_n$. In fact if $|\xi b - a| < |\xi k_n - h_n|$ for some $n \geq 0$, then prove that $b \geq k_{n+1}$.

Or

(b) Prove that any periodic simple continued fraction is a quadratic irrational number and conversely.

20. (a) The norm of a product equals the product equals the product of the norms, $N(\alpha\beta) = N(\alpha)N(\beta)$. $N(\alpha) = 0$ iff $\alpha = 0$. The norm of an integer in $\mathbb{Q}(\sqrt{m})$ is a rational integer. If γ is an integer in $\mathbb{Q}(\sqrt{m})$, then prove that $N(\gamma) = \pm 1$ iff γ is a unit.

Or

(b) Prove that the fields $\mathbb{Q}(\sqrt{m})$ for $m = -1, -2, -3, -7, 2, 3$, are Euclidean and so have the unique factorization property.

(6 pages) **Reg. No.: ....**

Code No. : 6364 **Sub. Code : ZMAM 11**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

First Semester

Mathematics — Core

ALGEBRA — I

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A homomorphism ϕ from G into $\overline{G}$ is said to be an isomorphism if ϕ is ————

   (a) one to one (b) unto

   (c) not one to one (d) bijective

2. Every subgroup of an abelian group is ————

   (a) right coset (b) last coset

   (c) normal (d) not normal

3. In a group $b^5 = e$ and $aba^{-1} = a^2$ for some $a, b \in G$. The order of a is ————

   (a) 5 (b) 10

   (c) 0 (d) divisor of 10

4. Let G be a group and ϕ an automorphism of G. If $a \in G$ is of order $o(a) > 0$, then $o(\phi(a)) =$ ————

   (a) 0 (b) 1

   (c) $o(a)$ (d) ∞

5. If $\alpha = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ then $\alpha\beta =$ ————

   (a) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$

   (c) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

6. If $o(G) = p^2$ where p is a prime number then G is ————

   (a) normal (b) left coset

   (c) right coset (d) abelian

7. The value of $9c_2$ is ————

   (a) 18 (b) 8

   (c) 32 (d) 36

8. The number of p-sylow subgroups in G, for a given prime is of the form ————

(a) $1+kp$ (b) $1-kp$

(c) kp (d) $\frac{1+k}{p}$

9. If $\phi \neq 1 \in \hat{G}$ where G is an abelian group then $\sum_{g \in G} \phi(g) =$ ————

(a) 1 (b) 2

(c) ∞ (d) 0

10. The number of non-isomorphic abelian groups of order p^n, p an prime, equals the number of partitions of ————.

(a) $\frac{n}{2}$ (b) $n!$

(c) n (d) $n-1$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If G is a finite group and N is a normal subgroup of G, then prove that $o(G/N) = o(G)/o(N)$.

Or

(b) If ϕ is a homomorphism of G into $\overline{G}$, then prove that :

(i) $\phi(e) = \overline{e}$, the unit element of $\overline{G}$.

(ii) $\phi(x^{-1}) = \phi(x)^{-1}$ for all $x \in G$

12. (a) Show that $\mathcal{I}(G) \approx G/Z$, where $\mathcal{I}(G)$ is the group of inner automorphisms of G, and Z is the center of G.

Or

(b) If H is a subgroup of G show that for every $g \in G$, gHg^{-1} is a subgroup of G.

13. (a) Prove that $N(a)$ is a subgroup of G.

Or

(b) If $o(G) = p^n$ where p is a prime number, then prove that $Z(G) \neq (e)$.

14. (a) Prove that $n(k) = 1 + p + ... + p^{k-1}$.

Or

(b) If $p^m \mid o(G), p^{m+1} \nmid o(G)$, then prove that G has a subgroup of order p^m.

15\. (a) Let G be a group and suppose that G is the integral direct production of $N_1, \ldots, N_n$. Let $T = N_1 \times N_2 \times \ldots \times N_n$. Then prove that G and T are isomorphic.

Or

(b) If G and G' are isomorphic abelian groups, then prove that for every integer s, $G(s)$, and $G'(s)$ are isomorphic.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16\. (a) State and prove Sylow's theorem for Abelian groups.

Or

(b) Let ϕ be a homomorphism of G onto $\overline{G}$ with kernel K, and let $\overline{N}$ be a normal subgroup of $\overline{G}, N = \{x \in G \mid \phi(x) \in \overline{N}\}$. Then prove that $G/N \approx \overline{G}/\overline{N}$. Equivalently. $G/N \approx (G/K)/(N/K)$.

17\. (a) If G *is* a group, then prove that $\mathcal{A}(G)$, the set of automorphisms of G, is also a group.

Or

(b) Let G be a finite group, T an automorphism of G with the property that $xT = x$ iff $x = e$. Suppose further that $T^2 = 1$ prove that G must be abelian.

18\. (a) State and prove Cauchy theorem.

Or

(b) Prove : $o(G) = \sum \frac{o(G)}{o(N(a))}$ where this sum runs over one element a in each conjugate class.

19\. (a) State and prove Sylow theorem.

Or

(b) Prove that S_{p^k} has a p-sylow subgroup.

20\. (a) Let G be an abelian group of order p^n, p a prime. Suppose that $G = A_1 \times A_2 \times \ldots \times A_k$, where each $A_i = (a_i)$ is cyclic of order p^{n_i}, and $n_1 \geq n_2 \geq \ldots \geq n_k > 0$. If m is an integer such that $n_t > m \geq n_{t+1}$ then prove that $G(p^m) = B_1 \times \ldots \times B_t \times A_{t+1} \times \ldots \times A_k$ where B_i is cyclic of order p^m, generated by $a_i^{p^{n_i - m}}$, for $i \leq t$. The order of $G(p^m)$ is p^u, where $u = mt\sum_{i=t+1}^{k} n_i$.

Or

(b) Show that the two abelian groups of order p^n are isomorphic iff they have the same invariants.

(8 pages) **Reg. No. :** ....

Code No. : 6365 **Sub. Code : ZMAM 12**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

First Semester

Mathematics — Core

ANALYSIS — I

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Any discrete metric space is ————

   (a) first category (b) second category

   (c) third category (d) none of these

2. Any discrete metric space having more than one point is ————

   (a) connected (b) finite

   (c) null set (d) disconnected

3. If the sequence $\{a_n\}$ is bounded and sequence $\{b_n\}$ converges to zero then the sequence $\{a_n b_n\}$ ————

   (a) diverges to $+\infty$ (b) diverges to $-\infty$

   (c) converges to zero (d) none of these

4. Find lim sup a_n for the sequence $\{a_n\} = \{n!\}$ ————

   (a) 1 (b) 0

   (c) ∞ (d) none of these

5. Applying Cauchy's root test the series $\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^n$ is ————

   (a) convergent

   (b) divergent

   (c) neither convergent nor divergent

   (d) both convergent and divergent

6. If the n^{th} term of a series is $a_n = \frac{1.2.3\cdots n}{3.5.7\cdots 2n-1}$ then $\lim_{n\to\infty}\frac{a_n}{a_{n+1}} =$ ————

   (a) 1 (b) 2

   (c) $\frac{1}{2}$ (d) 0

7. Which of the following is equivalent to compactness in a metric space M?

(a) M is totally bounded

(b) M is complete

(c) Every bounded subset of M has a limit point

(d) Every infinite subset of M has a limit point

8. Which of the following subset of R is both compact and connected? ————

(a) R (b) (0, 1)

(c) [0,100] (d) Q

9. Let f be defined on $[a, b]$; if f has a local maximum at a point $x \in (a,b)$, and if $f'(x)$ exists, then $f'(x)$ ————

(a) 1 (b) 2

(c) 0 (d) ∞

10. Suppose f is differentiable in (a, b) if $f'(x) = 0$ or all $x \in (a,b)$, then f is ————

(a) monotonically increasing

(b) monotonically decreasing

(c) constant

(d) none of these

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that the closed subsets of compact sets are compact.

Or

(b) Let K be a positive integer. If $\{I_n\}$ is a sequence of k-cells such that $I_n \supset I_{n+1} (n = 1, 2, 3, \cdots)$, then prove that $\bigcap_1^\infty I_n$ is not empty.

12. (a) Show that if $p > 1$, $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ converges; if $p \le 1$, the series diverges.

Or

(b) Suppose $\{S_n\}$ is monotonic. Then prove that $\{S_n\}$ converges iff it is bounded.

13. (a) If $\sum a_n = A$, and $\sum b_n = B$, then prove that $\sum (a_n + b_n) = A + B$, and $\sum ca_n = CA$ for any fixed c.

Or

(b) Prove :

(i) the partial sums A_n of $\sum a_n$ from a bounded sequence;

(ii) $b_0 \geq b_1 \geq b_2 \geq \cdots$;

(iii) $\lim_{n\to\infty} b_n = 0$.

14. (a) Let f be monotonic on (a, b). Then prove that the set of points of (a, b) at which f is discontinuous is at most countable.

Or

(b) Suppose f is a continuous mapping of a compact metric space X into a metric space Y then. Prove that $f(X)$ is compact.

15. (a) If f and g are continuous real function on $[a, b]$ which are differentiable in (a, b), then prove that there is a point $x \in (a, b)$ at which $[f(b) - f(a)]g'(x) = [g(b) - g(a)]f'(x)$. Note that differentiability is note required at the end points.

Or

(b) Suppose f is a real differentiable function on $[a, b]$ and suppose $f'(a) < \lambda < f'(b)$. Then prove that there is a point $x \in (a, b)$ such that $f'(x) = \lambda$. A similar result holds of course if $f'(a) > f'(b)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Prove that a subset E of the real line R^1 is connected iff it has the following property: If $x \in E, Y \in E$, and $x < z < y$, then $z \in E$.

Or

(b) Suppose $K \subset Y \subset X$. Then prove K is compact relative to X iff K is compact relative to Y.

17. (a) Prove that the following :

(i) If $\{p_n\}$ is a sequence in a compact metrix space X, then some sub-sequence of $\{p_n\}$ converges to a point of X.

(ii) Every bounded sequence in R^k contains a convergent subsequence.

Or

(b) Prove that e is irrational

18. (a) Suppose

(i) $\sum_{n=0}^{\infty} a_n$ converges absolutely

(ii) $\sum_{n=0}^{\infty} a_n = A$,

(iii) $\sum_{n=0}^{\infty} b_n = B$,

(iv) $C_n \sum_{k=0}^{n} a_k\, b_{n-k}\ (n = 0, 1, 2, \cdots)$.

Then prove that $\sum_{n=0}^{\infty} C_n = AB$. That is, the product of two convergent series converges, and to the right value, if at least one of the two series converges absolutely.

Or

(b) State and prove Ratio Test.

19. (a) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that f uniformly continuous on X.

Or

(b) Let X, Y, E, f, and p is a limit point of E. Then prove that $\lim_{x \to p} f(x) = q$ iff $\lim_{n \to \infty} f(p_n) = q$ for every sequence $\{p_n\}$ in E such that $p_n \neq p, \lim_{n \to \infty} p_n = p$.

20. (a) Suppose f is continuous on $[a, b]$, $f'(x)$ exists at some point $x \in [a, b]$, g is defined on an interval I which contains the range of f, and g is differentiable at the point $f(x)$. If $h(t) = g(f(t))\ (a \le t \le b)$, then prove that h is differentiable at x, and $h'(x) = g'(f(x))f'(x)$.

Or

(b) State and prove that L' Hospital's rule.

(7 pages) **Reg. No. :** ....

Code No.: 6366 **Sub. Code: ZMAM 13**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

First Semester

Mathematics – Core

ANALYTIC NUMBER THEORY

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $n \mid n$, then it is called ________ property of divisibility.
   (a) reflexive
   (b) symmetric
   (c) transitivity
   (d) linearity

2. The series $\sum_{n=1}^{\infty} 1/p_n$ is ________
   (a) converges (b) diverges
   (c) countable (d) uncountable

3. The value $\varphi(8) =$ ________
   (a) 1 (b) 2
   (c) 4 (d) 0

4. The notation for Mobius function is ________.
   (a) $\varphi(n)$ (b) $\pi(n)$
   (c) $f(n)$ (d) $\mu(n)$

5. The identity function $I(n) = [1/n]$ is ________.
   (a) not multiplicative
   (b) multiplicative
   (c) completely multiplicative
   (d) not complete

6. If any two functions f and g are multiplicative, then ________ multiplicative
   (a) fg (b) f/g
   (c) none of the above (d) both

7. The value of $\lim_{x\to\infty} \frac{\pi(x)\log x}{x} =$ ______.

(a) 1 (b) 2

(c) 4 (d) 0

8. The average order of $\Lambda(n)$ is ______

(a) 2 (b) 1

(c) 4 (d) 0

9. The upper bound of $\pi(n) =$ ______.

(a) $\frac{1}{6}\frac{n}{\log n}$ (b) $\frac{n}{\log n}$

(c) $6\frac{n}{\log n}$ (d) $2\frac{n}{\log n}$

10. The value of $\lim_{x\to\infty} \frac{\log x}{\log \pi(x)} =$ ______.

(a) 0 (b) 2

(c) 4 (d) 1

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that every integer $n > 1$ is either a prime number or a product of prime numbers.

Or

(b) State and prove division algorithm.

12. (a) Prove that $\varphi(mn) = \varphi(m)\varphi(n)(d, \varphi(d))$ where $d = (m, n)$. Also prove that $\varphi(a) \mid \varphi(b)$ if $a \mid b$.

Or

(b) State and prove Mobius inversion formula.

13. (a) Given f with (1) = 1. Then prove that f is multiplicative if and only if $f(p_1^{a_1}, p_2^{a_2}, \ldots, p_r^{a_r}) = f(p_1^{a_1})f(p_2^{a_2})\ldots f(p_r^{a_r})$ for all primes p_i and all integers $a_i \geq 1$.

Or

(b) State and prove Generalized inversion formula.

14. (a) If $x \geq 1$, then prove that $\sum_{n\leq x}\frac{1}{n} = \log x + C + 0\left(\frac{1}{x}\right)$. Also prove that $\sum_{n\leq x} n^{\alpha} = \frac{x^{\alpha+1}}{\alpha+1} + 0(x^{\alpha})$ if $\alpha \geq 0$.

Or

(b) For all $x > 1$, show that $\sum_{n\leq x}\varphi(n) = \frac{3}{\pi^2}x^2 + 0(x\log x)$.

15. (a) For all $x \geq 1$, prove that $\left|\sum_{n \geq x} \frac{\mu(n)}{n}\right| \leq 1$ with equality holding only if $x < 2$.

Or

(b) For $x \geq 2$, show that $\sum_{p \leq x} \left[\frac{x}{p}\right] \log p = x \log x + 0(x)$ where the sum is extended over all primes $\leq x$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) (i) State and prove Euclidean algorithm.

(ii) If $(a,b) = 1$, then prove that $(a^n, b^k) = 1$ for all $n \geq 1$, $k \geq 1$.

Or

(b) (i) Prove that $n^4 + 4$ is composite if $n > 1$.

(ii) Prove that every integer $n > 1$ can be represented as a product of prime factors in only one way, apart from the order of the factors.

17. (a) State and prove the product formula for $\varphi(n)$.

Or

(b) Define Mobius function and find the relationship between Mobius function and Euler totient function.

18. (a) Define Liouville's function and for every $n \geq 1$ and prove that $\sum_{d|n} \lambda(d) = \begin{cases} 1, & if\ n\ is\ a\ square \\ 0, & otherwise \end{cases}$.

Or

(b) State and prove Generalized Mobius inversion formula.

19. (a) Prove that the set of lattice points visible from the origin has density $6/\pi^2$.

Or

(b) For all $x \geq 1$ and $\alpha > 0$, $\alpha \neq 1$, prove that,

(i) $\sum_{n \leq x} \sigma_1(n) = \frac{1}{2}\zeta(2)x^2 + 0(x \log x)$

(ii) $\sum_{n \leq x} \sigma_\alpha(n) = \frac{\zeta(\alpha + 1)}{\alpha + 1} x^{\alpha+1} + 0(x^\beta)$ where $\beta = \max\{1, \alpha\}$.

20. (a) For $n \geq 1$, prove that the n^{th} prime p_n satisfies the inequality $\frac{1}{6} n \log n < p_n < 12\left(n \log n + n \log \frac{12}{e}\right)$.

Or

(b) Prove that the following relations are logically equivalent

(i) $\lim_{x\to\infty} \dfrac{\pi(x)\log x}{x} = 1$

(ii) $\lim_{x\to\infty} \dfrac{\vartheta(x)}{x} = 1$

(iii) $\lim_{x\to\infty} \dfrac{\psi(x)}{x} = 1$

Reg. No. : ....

Code No. : 6367 Sub. Code : ZMAM 14

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

First Semester

Mathematics — Core

OPERATIONS RESEARCH

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The unit "transportation" cost from period i to period j is computed as C_{ij} =

   (a) Production cost is i, $i = j$

   (b) Production cost is i + holding cost from i to j, $i < j$

   (c) Production cost is i + penality cost from to i to j, $i < j$

   (d) All the above

2. Which method yields the best starting solutions of the transportation problem

(a) North west-corner method

(b) Least-cost method

(c) Vogel approximation method

(d) None

3. A circuit is a loop in which all the branches are oriented in the ————.

(a) Opposite direct (b) Same direction

(c) Both direction (d) None

4. Total float of an activity is TF_{ij} = ————.

(a) $LC_j - ES_i - D_{ij}$ (b) $LC_j + ES_i - D_{ij}$

(c) $LC_j - ES_i + D_{ij}$ (d) none

5. Which one of the following is IP

(a) Zero-one (b) Mixed zero-one

(c) Pure integer (d) Mixed

6. Additive algorithms required presenting the 0–1 problem in a convenient form that satisfies ———— requirement

(a) 1 (b) 2

(c) 3 (d) none

7. During the classic EOQ model, the reorder point occurs when the ———— to LD units.

(a) inventory level drops

(b) inventory level increases

(c) all the above (a) and (b)

(d) none

8. In constant rate demand with instantaneous replenishment and no shortage model Y^* = ————

(a) $\frac{DK}{Y} + \frac{YK}{2}$ (b) $\sqrt{\frac{2DK}{h}}$

(c) $\frac{Yh}{2}$ (d) $\sqrt{2DKh}$

9. The expected waiting time in the model $(M/M/1)$: $(G_D/\infty/\infty)$ is

(a) $\frac{\rho}{1-\rho}$ (b) $\frac{1}{\mu(1-\rho)}$

(c) $\frac{\rho}{\mu(1-\rho)}$ (d) $\frac{\rho^2}{1-\rho}$

10. In $(M/M/\infty):(G_D/\infty/\infty)$ model $P_0 =$ ________.

(a) $1-\rho$ (b) $e^{-\rho}$

(c) $\frac{1}{\mu}$ (d) $\lambda\mu$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that in a symmetric TSP, any three distinct cost elements $C(i\ j)$, $C(j, k)$, $C(k,i)$ can be set to infinity without eliminating a minimum length tours.

Or

(b) Solve the 4-city TSP whose distance matrix is given in the following table :

City	1	2	3	4
1	–	12	10	14
2	12	–	13	8
3	10	13	–	12
4	14	8	12	–

12. (a) Explain Maximal Flow algorithm.

Or

(b) Explain Dijkstra's algorithm.

13. (a) Solve the following zero-one problem using implicit enumeration algorithm.

Maximize $4x_1 + 3x_2 - 2x_3$

Subject to

$x_1 + x_2 + x_3 \le 8$

$2x_1 - x_2 - x_3 \le 4$

$x_2, x_3 = 0, 1$

Or

(b) Solve the MILP :

Maximize $2x_1 + 3x_2$

Subject to

$3x_1 + 4x_2 \le 10$

$x_1 + 3x_2 \le 7$

$x_1 \ge 0$

$x_2 \ge 0$

and are integers.

14. (a) An item is consumed at the rate of 30 items per day. The holding cost per unit per day is \$. 05 and the set up cost in \$ 100. Suppose that no shortage is allowed and that the purchasing cost per unit is \$10 for any quantity not exceeding 500 units and \$ 8 otherwise. Determine the optimal inventory policy given a 21-day lead time.

Or

(b) A music store sells a best-selling compact disc. The daily demand for the disc in approximately normally distributed with mean 200 disc and a standard deviation of 20 disc. The cost of keeping the disc in the store is $. 04 per disc per day. It cost the store $ 100 to place a new order. The supplier normally specifies a 7-day lead time for delivery. Assuming that the store wants so limit the probability of running out of disc during the lead time to no more than .02, determine the stores optimal inventory policy.

15. (a) Consider the production - consumption inventory model with back orders. The data are D = 10000 / year, P = 16000 / year, C_0 = 350 / set up, C_c = 3.6 / unit / year and D = 100 unit / year. Find the batch quantity Q and the total cost.

Or

(b) Explain the model $(M/M/1):(GD/\infty/\infty)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve the transportation model.

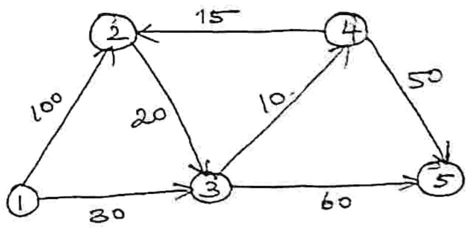
$ 0	2	1	6
2	1	5	7
2	4	3	7
5	5	10	

Or

(b) Solve the assignment model

1	4	6	3
9	7	10	9
4	5	11	7
8	7	8	5

17. (a) D Find the shortest path from 1 to 5 using Dijkstra's algorithm.



Or

(b) Discuss the computations of critical path method.

18. (a) Solve the following 0-1 problem

Maximize $w = 3y_1 + 2y_2 - 5y_3 - 2y_4 + 3y_5$

Subject to

$y_1 + y_2 + y_3 + 2y_4 + y_5 \le 4$

$7y_1 + 3y_3 - 4y_4 + 3y_5 \le 8$

$11y_1 - 6y_2 + 3y_4 - 3y_5 \ge 3$

$y_1, y_2, y_3, y_4, y_5 = (0, 1)$

Or

(b) Solve the following by using fractional cut

Maximize $z = 3x_1 + x_2 + 3x_3$

Subject to :

$x_1 + 2x_2 + x_3 \le 4$

$4x_2 - 3x_3 \le 2$

$x_1 - 3x_2 + 2x_3 \le 2$

$x_1, x_2, x_3 \ge 0$

and are integers.

19. (a) The following data describe four inventory items. The company wishes to determine the economic order quantity for each of the four items such that the total number of orders per year (365 days) is atmost 150.

Item	K_i \$	D_i units/d	h_i
1	100	10	.1
2	50	20	2
3	90	5	.2
4	20	10	.1

Or

(b) Explain the model Multi-item with storage Limitations.

20. (a) Explain $(M/M/C):(GD/N/\infty)$, $C \le N$.

Or

(b) Patients arrive at a clinic according to a Poisson distribution at the rate of 20 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patients is exponential with mean of 8 minutes

(i) What is the probability that an arriving patient will not wait?

(ii) What is the probability that a patient to find a vacant seat in the room?

(iii) What is the expected waiting time until a patient leaves the clinic?

Reg. No. : ....

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Code No. : 6368 **Sub. Code : ZMAM 15**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

First Semester

Mathematics – Core

ORDINARY DIFFERENTIAL EQUATION

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Which of the following second order non-homogenous linear differential equation?

(a) $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y^2=R(x)$

(b) $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y^2=0$

(c) $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=R(x)$

(d) $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=0$

2. Any linear combination of two solution of the homogeneous equation is ————

(a) Not a solution (b) Solution

(c) Linear (d) Non-linear

3. If $f(x)$ and $g(x)$ are analytic at x_0, then ———— is analytic.

(a) $f(x)+g(x)$ (b) $f(x)g(x)$

(c) $\frac{f(x)}{g(x)}$ if $g(x_0)\neq 0$ (d) All the above

4. The series $\sum_{n=0}^{\infty}\frac{x^n}{n!}$ ————

(a) converges for all x

(b) converges only at $|x|<1$

(c) diverges at $|x|>1$

(d) diverges for all $x\neq 0$

5. Which of the following series is Frobenius series?

(a) $y=a_0+a_1x^m+a_2x^{(m+1)}+\ldots$

(b) $y=a_0x^m+a_1x^{m+1}+a_2x^{m+2}+\ldots$

(c) $y=x^m(a_0+a_2+a_1+\ldots)$

(d) $y=x^m(a_0x+a_1x^2+a_2x^3+\ldots)$

6. If the function $(x-x_0)P(x)$ and $(x-x_0)^2Q(x)$ are not analytic then singular point x_0 is said to be ————

(a) Irregular (b) Regular

(c) Singular (d) Non-Singular

7. $J_{\frac{-1}{2}}(x)=$

(a) $\sqrt{\frac{2}{\pi x}}\cos x$ (b) $\sqrt{\frac{1}{\pi x}}\cos x$

(c) $\sqrt{\frac{2}{\pi x}}\sin x$ (d) $\sqrt{\frac{1}{\pi x}}\sin x$

8. $\Gamma(p)=$

(a) $\Gamma(p+1)$ (b) $\Gamma(p-1)$

(c) $\frac{\Gamma(p+1)}{p}$ (d) $p\Gamma(p+1)$

9. The value of $\begin{vmatrix} e^{3t} & e^{2t} \\ e^{3t} & 4e^{2t} \end{vmatrix}$

(a) e^{4t} (b) $3e^{5t}$

(c) $2e^{5t}$ (d) $3e^{4t}$

10. The auxiliary question of $\begin{cases}\frac{dx}{dt}=3x-4y\\ \frac{dy}{dt}=x-y\end{cases}$ is

(a) $(m-1)^2=0$ (b) $(m+1)^2=0$

(c) $m^2+2m-1=0$ (d) $m^2-2m-1=0$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that if $y_1(x)$ and $y_2(x)$ are any two solution of $y''+P(x)y'+Q(x)y=0$, then $c_1y_1(x)+c_2y_2(x)$ is also a solution for any constants c_1 and c_2.

Or

(b) Show that $y=c_1x+c_2x^2$ is the general solution of $x^2y''-2xy'+2y=0$ on any interval not containing 0, and find the particular solution for which $y(1)=3$ and $y'(1)=5$.

12. (a) Find the general solution of $(1+x^2)y''+2xy'-2y=0$ in terms of power series in x.

Or

(b) Find the power series for $\frac{1}{(1-x)^2}$ by squaring.

[P.T.O.]

13. (a) Find two independent Frobenius series solutions of $x^2y'' - x^2y' + (x^2 - 2)y = 0$.

Or

(b) Determine the nature of the point $x = 0$ for $y'' + (\sin x)y = 0$.

14. (a) Show that $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$.

Or

(b) Prove that $\frac{d}{dx}[x^pJ_p(x)] = x^pJ_{p-1}(x)$.

15. (a) Show that $\begin{cases} x = e^{4t} \\ y = e^{4t} \end{cases}$ and $\begin{cases} x = e^{-2t} \\ y = -e^{-2t} \end{cases}$ are solutions of the homogeneous system

$$\begin{cases} \frac{dx}{dt} = x + 3y \\ \frac{dy}{dt} = 3x + y \end{cases}.$$

Or

(b) Show that $\begin{cases} x = 2e^{4t} \\ y = 3e^{4t} \end{cases}$ and $\begin{cases} x = e^{-t} \\ y = -e^{-t} \end{cases}$ are solutions of the homogeneous system

$$\begin{cases} \frac{dx}{dt} = x + 2y \\ \frac{dy}{dt} = 3x + 2y \end{cases}.$$

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) Show that $y = c_1 \sin x + c_2 \cos x$ is the general solution of $y'' + y = 0$ on any interval, and find the particular solution for which $y(0) = 2$ and $y'(0) = 3$.

Or

(b) Find the differential equation of $y = c_1 \sin kx + c_2 \cos kx$ and $y = c_1 + c_2 e^{-2x}$ by eliminating the constants c_1 and c_2.

17. (a) Derive Binomial Series expansion.

Or

(b) Let x_0 be an arbitrary point of the differential equation $y'' + P(x)y' + Q(x)y = 0$, and let a_0 and a_1 be arbitrary constants. Then prove that there exists a unique function $y(x)$ that is analytic at x_0, is a solution of the equation in a certain neighborhood of this point and satisfies the initial conditions $y(x_0) = a_0$ and $y'(x) = a_1$. Furthermore, prove that if the power series expansion of $P(x)$ and $Q(x)$ are valid on an interval $|x - x_0| < R, R > 0$, then the power series expansion of this solution is also valid on the same interval.

18. (a) Verify that the origin is a regular singular point and calculate two independent Frobenius series solution for $2xy'' + (3 - x)y' - y = 0$.

Or

(b) Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & if\ m \neq n \\ \frac{2}{2n+1} & if\ m = n \end{cases}$, where $P_0(x), P_1(x), P_2(x), \ldots, P_n(x)$ is asequence of orthogonal functions on the interval $-1 \le x \le 1$.

19. (a) Find the value of $\Gamma\left(\frac{1}{2}\right)$.

Or

(b) Prove that

$$\int_0^1 xJ_p(\lambda_m x)J_p(\lambda_n x)dx = \begin{cases} 0 & if\ \ m \neq n \\ \frac{1}{2}J_{p+1}(\lambda_n)^2 & if\ \ m = n \end{cases}$$

20. (a) Find the General solution of $\begin{cases} \frac{dx}{dt} = 7x + 6y \\ \frac{dy}{dt} = 2x + 6y \end{cases}$.

Or

(b) Find the General solution of $\begin{cases} \frac{dx}{dt} = -4x - y \\ \frac{dy}{dt} = x - 2y \end{cases}$.

(7 pages) **Reg. No. :** ....

Code No. : 6369 **Sub. Code : ZMAM 21**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Second Semester

Mathematics – Core

ALGEBRA – II

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Let J be the ring of integers, J_n, the ring of integers modulo n. Define $\phi : J \to J_n$ by $\phi(a)$ = remainder of a on division by n. Its Kernel $I(\phi)$ is

   (a) $\{0\}$

   (b) J

   (c) the set of all multiples on n

   (d) $\{0,1,2, n-1\}$

2. Which one of the following is a maximal ideal of the ring of integers?

   (a) (60) (b) (13)

   (c) (2022) (d) (15)

3. A solution of the congruence $x^2 \equiv -1 (\text{mod } 13)$ is

   (a) 3 (b) 0

   (c) 5 (d) 6

4. A necessary and sufficient condition that the element a is the Euclidean ring be a unit is that

   (a) $d(a) = 1$

   (b) $d(a) = d(1)$

   (c) $d(a) \mid d(1)$

   (d) a is a prime element

5. Which one of the following is a primitive polynomial

   (a) $2 + 4x^2 + 8x^5$

   (b) $5 + 10x^2 + 15x^4$

   (c) $20x^4 + 15x^3 + 10x^2 + 1$

   (d) $2 + 2x + 2x^2$

6. Which one of the following is not true the polynomial $x^2 + 1$ is irreducible over?

(a) the complex field

(b) the real field

(c) the integers mod 3

(d) the field of rational numbers

7. In the ring of integers z, $\sqrt{(180)}$ is

(a) (4.9.5) (b) (2.3.5)

(c) (2.3.10) (d) (3.5.7)

8. In any ring R, an element $a \in rad\, R$ if and only if

(a) $1 - ra$ is invertible for each $r \in R$

(b) ra is invertible for each $r \in R$

(c) $1 - ra$ is invertible for some $r \in R$

(d) ra is invertible for some $r \in R$

9. b is a quasi-inverse of a if

(a) $ab = 1$ (b) $a + b - ab = 0$

(c) $a + b = 0$ (d) $a - b - ab = 0$

10. A ring R is isomorphic to a subdirect sum of integral domains if and only if R is

(a) semi simple

(b) simple

(c) without prime radical

(d) an integral domain

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If F is a field, prove its only ideals are (0) and F itself.

Or

(b) If ϕ is a homomorphism of R into R', prove that $\phi(0) = 0$ and $\phi(-a) = -\phi(a)$ for every $a \in R$.

12. (a) Let R be a Euclidean ring and let A be an ideal of R. Prove that there exists an elements $a_0 \in A$ such that A consists exactly of all $a_0 x$ as x ranges over R.

Or

(b) Prove that $J[i]$ is a Euclidean ring.

13. (a) If $f(x), g(x)$ are two non zero elements of $F[x]$, prove that $\deg(f(x)g(x)) = \deg f(x) + \deg g(x)$.

Or

(b) State and prove Gauss' lemma.

14. (a) Let I be an ideal of the ring R. Prove that $I \subseteq rad\, R$ if and only if each element of the coset $1+I$ has an inverse in R.

Or

(b) For any ring R, prove that the quotient ring $R \mid Rad\, R$ is without prime radical.

15. (a) Define the J-radical $J(R)$ of a ring and a J-semi simple ring. Prove that the ring of even integers is J-semi simple.

Or

(b) Prove that a ring R is isomorphic to a sub direct sum of ring R_i, if and only if R contains a collection of ideals $\{I_i\}$ such that $R/I_i \simeq R_i$ and $\bigcap I_i = \{0\}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If U is an ideal of the ring R, prove that $R \mid U$ is a ring and is a homomorphic image of R.

Or

(b) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Prove that R is a field.

17. (a) Prove that the ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring R is and only if a_0 is a prime element of R.

Or

(b) Let p be a prime integer and suppose that for some integer c relatively prime to p we can find integers x and y such that $x^2 + y^2 = cp$. Prove that p can be written as the sum of squares of two integers.

18. (a) Define the sum $p(x)+q(x)$ and product $p(x)\cdot q(x)$ of two polynomials and state and prove the division algorithm for polynomials.

Or

(b) State and prove the Eisenstein criterion.

19. (a) If I is an ideal of the ring R, prove that

(i) $rad(R/I) \supseteq \dfrac{radR + I}{I}$ and

(ii) Whenever $I \subseteq radR$, $rad(R/I) = (rad\, R)/I$

Or

(b) (i) For any ring R, prove that $radR[x] = RadR[x]$.

(ii) Let e and e' be two idempotent elements of the ring R such that $e - e' \in Rad\,R$ prove that $e = e'$.

20. (a) If R is a ring such that $J(R) \neq R$, then prove that $J(R) = \bigcap\{M \mid M$ is a modular maximal ideal of $R\}$.

Or

(b) Prove that a ring R is isomorphic to a sub direct sum of fields it and only if for each non zero ideal I of R, then exists an ideal $J \neq R$ such that $I + J = R$.

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Code No. : 6370 **Sub. Code : ZMAM 22**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022

Second Semester

Mathematics – Core

ANALYSIS – II

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Given two partitions, P_1 and P_2. we say that P^* is their common refinement if P^* = ————

   (a) $P_1 \cap P_2$ (b) $P_1 - P_2$

   (c) $P_1 \cup P_2$ (d) $P_1 + P_2$

2. $\int_{-a}^{b} f\,d\alpha$ ———— $\int_{b}^{-b} f\,d\alpha$

   (a) $\geq$ (b) $\leq$

   (c) $=$ (d) $\neq$

3. If γ is Rectifiable then $\wedge(\gamma) <$ ————

   (a) 0 (b) 1

   (c) 2 (d) ∞

4. If x is irrational then the value of $\lim_{m\to\infty}\lim_{n\to\infty}(\cos m!\pi x)^{2n}$ is ————

   (a) 0 (b) 1

   (c) $-\infty$ (d) ∞

5. The value of $\int_0^{2\pi}(\sin n_k x - \sin n_{k+1}x)^2\,dx =$ ————

   (a) 0 (b) π

   (c) 2π (d) ∞

6. If $\{f_n\}$ is uniformly bounded on E if there exists a number M such that $|f_n(x)|$ ————M $(x \in E, n = 1,2,3,...)$

   (a) $<$ (b) $\leq$

   (c) $>$ (d) $\geq$

7. The value of $\lim_{x\to 0}\dfrac{\log(1+x)}{x}$ is ————

   (a) 1 (b) 0

   (c) e (d) e^x

8. The value of $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$ is ____________

(a) 1 (b) 0

(c) e (d) e^x

9. The value of $\int_0^\infty \left(\frac{\sin x}{x}\right)^2 dx$ is ____________

(a) $\frac{\pi}{2}$ (b) π

(c) 2π (d) $\sqrt{\pi}$

10. The value of $\Gamma\left(\frac{1}{2}\right)$ is ____________

(a) 0 (b) $\frac{1}{2}$

(c) $\sqrt{\pi}$ (d) π

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove if $f \in \mathscr{R}$ on $[a,b]$ and if there is a differentiable function F on $[a,b]$ such that $F' = f$ then $\int_a^b f(x)\,dx = F(b) - F(a)$.

Or

(b) If f is monotonic on $[a,b]$, and α is continuous on $[a,b]$, then prove that $f \in \mathscr{R}(\alpha)$.

12. (a) Suppose K is compact, and Prove that the following:

(i) $\{f_n\}$ is a sequence of continuous functions on K,

(ii) $\{f_n\}$ converges point wise to a continuous function f on K

(iii) $f_n(x) \geq f_{n+1}(x)$ for all $x \in K$, $n = 1,2,3,...$ Then $f_n \to f$ uniformly on K.

Or

(b) If f maps $[a,b]$ into R^k and if $f \in \mathscr{R}(\alpha)$ for some monotonically increasing function α on $[a,b]$, then Prove that, $|f| \in \mathscr{R}(\alpha)$, and $\left|\int_a^b f\,d\alpha\right| \leq \int_a^b |f|\,d\alpha$.

13. (a) If K is compact, if $f_n \in \mathscr{C}(K)$ for $n = 1,2,3,...,$ and if $\{f_n\}$ is point wise bounded and equicontinuous on K, then Prove that $\{f_n\}$ is uniformly bounded on K.

Or

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Code No. : 6371 **Sub. Code : ZMAM 23**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Second Semester

Mathematics — Core

ADVANCED CALCULUS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. Let D be the set of points (x, y) with $0 \leq x \leq 1$, $0 \leq y \leq 1$ and both x and y rational. Then

   (a) $\underline{A}(D) = \overline{A}(D) = 0$

   (b) $\underline{A}(D) = 1, \overline{A}(D) = 0$

   (c) $\underline{A}(D) = 0, \overline{A}(D) = 1$

   (d) $\underline{A}(D) = \overline{A}(D) = 1$

2. Let D be the region between the line $y = x$ and the parabola $y = x^2$. Take $f(x, y) = xy^2$. Then $\iint_D f$ is

(a) $\frac{1}{40}$ (b) $\frac{1}{20}$

(c) $\frac{1}{80}$ (d) $\frac{1}{10}$

3. Let $S: \begin{cases} u = x + y \\ v = x - y \\ w = x^2 \end{cases}$ the image of (1, 2) under S is

(a) (3, 1, 1) (b) (1, 1, 0)

(c) (3, –1, 1) (d) (3, –1)

4. Let $T: \begin{cases} u = x^2 + y - z \\ v = xyz^2 \\ w = 2xy - y^2 z \end{cases}$ Then $dT|_{(1,1,1)}$ is

(a) $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 2 \\ 2 & 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 & -1 \\ 2 & 0 & -1 \\ 1 & 1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -1 & 1 \\ 1 & 1 & 2 \\ 2 & 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

5. The Jacobian of the transformation

$T: \begin{cases} u = x \cos y \\ v = x \sin y \end{cases}$ is

(a) $x^2 \sin y$

(b) x

(c) $x \cos y$

(d) x^2

6. The sine and cosine functions are

(a) both linearly dependent and functionally dependent

(b) linearly independent and functionally dependent

(c) linearly dependent but not functionally dependent

(d) linearly independent but not functionally dependent

7. If γ is a smooth curve whose domain is the interval $[a, b]$ then $L(\gamma)$ is given by

(a) $\int_a^b \gamma'(t)\, dt$ (b) $\int_a^b \sqrt{|\gamma'(t)|}\, dt$

(c) $\int_a^b |\gamma'(t)|\, dt$ (d) $\int_a^b |\gamma'(t)|^2\, dt$

8. If γ is a curve of class C'' with arc length as the parameter, then the curvature of γ at the point corresponding to $t = c$ is

(a) $k = |\gamma''(c)|$ (b) $k = |\gamma'(c)|$

(c) $k = \gamma''(c)$ (d) $k = \gamma'(c)$

9. If ω is any differential form of class C'', then ddw is

(a) w (b) 0

(c) $-w$ (d) w^*

10. If $x = u^2 + v$, $y = v$ and $\sigma = xy^2 dxdy$ then σ^* is

(a) $(u^3v^2 + uv^3)dudv$

(b) $2uv^2 dudv$

(c) $(2u^3v^2 + 2uv^3)dudv$

(d) $2uv^3 dudv$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Let f and g be continuous and bounded on D prove that $\iint_D (f+g) = \iint_D f + \iint_D g$ and $\iint_D Cf = C\iint_D f$ for any constant C.

Or

(b) Show that for $x > 0$

$$\int_0^{\frac{\pi}{2}} \log[\sin^2\theta + x^2\cos^2\theta]d\theta = \pi\log\left(\frac{x+1}{2}\right)$$

12. (a) Consider the following linear transformations of the plane into itself.

$$S:\begin{cases} u = 2x - 3y \\ v = x + y \end{cases} \quad T:\begin{cases} u = x + y \\ v = 3x + y \end{cases}$$

Find ST and TS and check whether $ST = TS$ or not.

Or

(b) For any $P \in S$ and any $u \in R^3$, prove that $dg|_p(u) = Dg(p) \cdot u$, where g is a real valued function of class C' defined on an open set S in 3-space.

13. (a) Let T be a transformation from R^n into R^n which is of class C' in an open set D and suppose that $J(P) \neq 0$ for each $P \in D$. Prove that T is locally 1-to-1 in D.

Or

(b) Let F and G be of class C' in an open set $D \subset R^5$. Let $p_0 = (x_0, y_0, z_0, u_0, v_0)$ be a point of D at which both of the equations. $F(x, y, z, u, v) = 0$, $G(x, y, z, u, v) = 0$ are satisfied. Suppose also that $O(F, G)/O(u, v) \neq 0$ at p_0. Prove that there are two function ϕ and ψ of class C' in a neighborhood N of (x_0, y_0, z_0) such that $u = \phi(x, y, z)$, $v = \psi(x, y, z)$ is a solution of $F = G = O$ in N giving u_0 and v_0 at (x_0, y_0, z_0).

14. (a) If E is a closed bounded subset of Ω of zero volume, prove that $T(E)$ has zero volume.

Or

(b) Let γ_1 and γ_2 be smoothly equivalent smooth curves, and let p be a simple point on their trace. Prove that γ_1 and γ_2 have the same direction at p.

15. (a) Let $\overline{a} = 2i - 3j + k, \overline{b} = i - j + 3k$, $\overline{c} = i - 2j$. Compute the vectors $(\overline{a} \times \overline{b}) \cdot \overline{c}$ and $a \times (b \times c)$.

Or

(b) If ω is any differential form of class C'', prove that $dd\omega = 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If f is continuous on R, prove that $\iint_R f$ exists.

Or

(b) Let R be the rectangle described by $a \le x \le b, c \le y \le d$ and let f be continuous on R. Prove that $\iint_R f = \int_a^b dx \int_c^d f(x, y)dy$.

17. (a) Define a linear transformation. Let L be a linear transformation from R^n into R^m represented by the matrix $[a_{ir}]$. Prove that there is a constant B such that

$|L(P)| \le B|P|$ for all points P.

Or

(b) Let T be differentials on an open set D and let S be differentiate on an open set containing $T(D)$. Prove that ST is differentiable on D and if $P \in D$ and $q = T(P)$ then $d(ST)|_p = dS|_q \, dT|_p$.

18. (a) Let T be of class C' on an open set D in n space, taking values in n space. Suppose that $J(P) \ne r$ for all $P \in D$. Prove that $T(D)$ is an open set.

Or

(b) Prove that the local inverses are themselves differentiable transformations and find a formula for their differentials.

19. (a) If γ is a smooth curve whose domain is the interval $[a, b]$, prove that γ is rectifiable and also show that $L(\gamma) = \int_a^b |\gamma'(t)| dt$.

Or

(b) Let F be an additive set function, defined on $\mathfrak{S}$ and a.c. suppose also that F is differentiable everywhere, and uniformly differentiable on compact sets, with the derivative a point function f. Prove that f is continuous everywhere and $F(s) = \iint_s f$ holds for every rectangle S.

20. (a) Let D be a closed convex region in the plane and let $w = A(x, y)\,dx + B(x, y)\,dy$ with A and B of class C' in D. Prove that $\iint_{\partial D} Adx + Bdy = \iint_{D} dw = \iint_{D} \left(\frac{\partial B}{\partial x} - \frac{\partial A}{\partial y}\right) dxdy$.

Or

(b) If α is a k form and β any differential form, prove that $d(\alpha\beta) = (d\alpha)\beta + (-1)^k \alpha(d\beta)$.

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Code No. : 5673 **Sub. Code : ZMAM 23**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2022

Second Semester

Mathematics — Core

ADVANCED CALCULUS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Let f and g be continuous and bounded on D, then If $F(p) \geq 0$ for all $p \in D$, $\iint_D f$ ———— 0.

   (a) $=$ (b) $>$

   (c) $\geq$ (d) $\leq$

2. If f is continuous on R, then $\lim_{d(N)\downarrow 0}|\overline{S}(N) - \underline{S}(N)|$ ———— 0.

(a) $\neq$ (b) $=$

(c) $>$ (d) $<$

3. The linear function L such that $L(1,0,0)$, $(0,1,0)$, $(0,0,1)$ is ————

(a) [2, 1, 3] (b) [–2, 1, 3]

(c) [2, –1, –3] (d) [2, –1, 3]

4. The differentials of the following transformations at the indicated points

$$\begin{cases} u = x + 6y \\ v = 3xy \\ w = x^2 - 3y^2 \end{cases}$$ at (1, 1) is ————

(a) $\begin{bmatrix} 1 & 6 \\ 3 & 3 \\ -2 & -6 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 6 \\ 3. & 3 \\ 2 & -6 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 6 \\ 3 & 3 \\ 2 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -6 \\ 3 & 3 \\ 2 & 6 \end{bmatrix}$

(b) Prove that let R be a cube in (x, y, z) space with faces parallel to the coordinate planes. Let ω be a 2-form $\omega = A\,dydz + B\,dzdx + C\,dxdy$.

---

5. Find the product of the matrices $\begin{bmatrix} \cos y & \sin y \\ -x^{-1}\sin y & x^{-1}\cos y \end{bmatrix} \begin{bmatrix} \cos y & -x\sin y \\ \sin y & x\cos y \end{bmatrix}$ is ————

   (a) 1 (b) $2I$

   (c) 0 (d) $-I$

6. Find the det of $\begin{bmatrix} 8 & 2 \\ 12 & 3 \end{bmatrix}$ is ————

   (a) 24 (b) –12

   (c) 0 (d) 12

7. If $T: \begin{cases} x = u + v \\ y = v - u^2 \end{cases}$ then the Jacobian is ————

   (a) $1 - 2u$ (b) $1 + 2u$

   (c) $1 + 2v$ (d) $1 - 2v$

8. If E is a closed bounded subset of Ω of zero volume, then $T(E)$ has ———— volume.

   (a) 0 (b) 1

   (c) –1 (d) ∞

9. If f is a scalar function of class C'', then $curl\ (grad(f)) =$

(a) 0 (b) 1

(c) −1 (d) ∞

10. If ω is any differential form of class C'', then $dd\omega =$ ————

(a) 0 (b) 1

(c) ∞ (d) −1

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let f and f_1 be defined and continuous for $x \in [a, b]$, $y \in [c, d]$ and F defined by $f(x) = \int_c^d f(x, y)\, dy$. Then, prove that $F'(x)$ exists on the interval $[a, b]$ and is given by $F'(x) = \int_c^d \frac{\partial f}{\partial x} dy = \int_c^d f_1(x, y)\, dy$.

Or

(b) Let B be the closed ball in n space, center 0, radius r, let T be a C' transformation defined on an open set containing B on which its Jacobian $J(p)$ never vanishes Suppose also that T is closed to the identify map, meaning that there is a number ρ such that

$0 < \rho < \frac{1}{2}$ and $|T(p) - p| \le pr$ for all $\rho \in \beta$.

Then, prove that T maps B onto a set $T(B)$ that contains all the points in the open ball centered at 0 of radius $(1 - 2\rho)r$.

20. (a) Prove that let D be a closed convex region in the plane, and let $\omega = A(x, y)dx + B(x, y)dy$ with A and B of class C' and D. then, $\int_{\partial D} A\, dx + B\, dy = \iint_D d\omega = \iint_D \left(\frac{\partial B}{\partial x} - \frac{\partial A}{\partial y}\right) dxdy$.

Or

(b) Prove that let T be continuous on a set D. then, any compact set $C \subset D$ is carried by T into a compact set $T(C)$, and any connected set $S \subset D$ is carried into a connected set $T(S)$.

(a) Prove that let T be of class C' in an open set D, with $J(p) \neq 0$ for all $p \in D$. Suppose also that T is globally 1-to-1 in D, so that there is an inverse transformation T^{-1} defined on the set $T(D) = D^*$. Then, T^{-1} is of class C' on D^*, $d(T^{-1})|_q = (dT|_p)^{-1}$, where $q = T(p)$.

Or

(b) If u, v and w are C' functions of x, y, and z in D, and if $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$ at all points of D, then u, v and w are functionally related in D. Find this relationship.

(a) Let T be a transformation from R^2 into R^2 which is of class C' in an open region D. furthermore, let T be conformal and have a strictly positive Jacobian throughout D. Then, prove that at each point of D, the differential of T has a matrix representation of the form $\begin{bmatrix} A & B \\ -B & A \end{bmatrix}$.

Or

(b) If f is continuous on R, then prove that $\lim_{d(N)\downarrow 0}\left|\overline{S}(N) - \underline{S}(N)\right| = 0$.

12. (a) Let the transformation S be continuous on a set A and T be continuous on a set B, and let $p_0 \in A$ and $S(p_0) = q_0 \in B$. Then, prove that the product transformation TS, defined by $TS(p) = T(S(p))$, is continuous at p_0.

Or

(b) Compute the rank of matrix $\begin{bmatrix} 3 & -6 & 9 \\ 2 & -4 & 6 \\ -2 & 4 & -12 \end{bmatrix}$.

13. (a) Compute the Jacobians transformation
$$\begin{cases} u = e^x \cos y \\ v = e^x \sin y. \end{cases}$$

Or

(b) Prove that, if T is continuous and 1-to-1 on a compact set D, then T has a unique inverse T^{-1} which maps $T(D) = D^*$ 1-to-1 onto D, and T^{-1} is continuous on D^*, for, the graph of T^{-1} is just the reflection of the graph of T and is also compact, so that the transformation T^{-1} must also be continuous.

14. (a) If E is a closed bounded subset of Ω of zero volume, then prove that $T(E)$ has zero volume.

Or

(b) If γ_1 and γ_2 are smoothly equivalent curves, then prove that $L(\gamma_1) = L(\gamma_2)$.

15. (a) If ω is any differential form of class C'', then prove that $dd\omega = 0$.

Or

(b) Prove that let T be a transformation of class C'' defined by $x = \phi(u, v), y = \psi(u,v)$, mapping a compact set D onto D^*. we assume that D and D^* are finite unions of standard region and that T is 1-to-1 on the boundary of D and maps it onto the boundary of D^*. let f be continuous in D^*. then $\iint_{D^*} f(x,y)\, dxdy = \iint_D f(\phi(u,v), \psi(u,v)) \frac{\partial(x,y)}{\partial(u,v)} dudv$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let ϕ' exist and be continuous on the interval $[\alpha, \beta]$ with $\phi(\alpha) = 0$ and $\phi(\beta) = b$. Let f be continuous at all points $\phi(u)$ for $\alpha \le u \le \beta$. Then, prove that $\int_a^b f(x)\,dx = \int_\alpha^\beta f(\phi(u))\phi'(u) du$.

Or

(b) Prove that let R be a closed rectangle, and let f be bounded in R and continuous at all points of R except those in a set E of zero area. Then $\iint_R f$ exists.

17. (a) Prove that let T be differentiable on an open set D, and let S be differentiable on D, and if $p \in D$ and $q = T(p)$, then prove that $d(ST)|_p = dS|_q\, dT|_p$.

Or

(7 pages) Reg. No. : ....

Code No. : 6372 **Sub. Code : ZMAM 24**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Second Semester

Mathematics – Core

DIFFERENTIAL GEOMETRY

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)
Answer ALL questions.

Choose the correct answer :

1. The arc length from a to any point u is given by

(a) $r = S(u) = \int_a^u R(u)du$

(b) $r = S(u) = \int_a^u |\dot{R}(u)|du$

(c) $r = S(u) = \int_a^u |R(u)|du$

(d) $r = S(u) = \int_a^u |\dot{R}(u)|^2 du$

2. The line of intersection of the nominal plant and the osculating plane at P is called

(a) the principal normal

(b) the principal tangent

(c) the principal curvature

(d) the principal line

3. The radius of spherical curvature is given by

(a) $R = \sqrt{\rho^2 + \sigma^2\rho^2}$ (b) $R = \sqrt{\rho^2 + \rho'^2}$

(c) $R = \sqrt{\rho^2 + \sigma^2\rho'^2}$ (d) $R = \sqrt{\rho^2 + \sigma^2}$

4. The equation of the involute is

(a) $R = r + t$ (b) $R = r + \rho n + \sigma\rho' b$

(c) $[R - r, \dot{r}, \ddot{r}] = 0$ (d) $R = r + (c - s)t$

5. For the paraboloid $x = u$, $y = v$, $z = u^2 - v^2$, F is

(a) $1 + 4u^2$ (b) $1 + 4v^2$

(c) $4uv$ (d) $-4uv$

6. The two parametric curves through a point P are orthogonal if at P

(a) $r_1 \times r_2 = 0$ (b) $r_1 \cdot r_2 = 0$

(c) $r_1 + r_2 = 0$ (d) $r_1 \times r_2 \neq 0$

7. When $v = c$ for all values of u, a necessary and sufficient condition that the curve $v = c$ is a geodesic is

(a) $EE_2 + FE_1 + 2EF_1 = 0$

(b) $EE_2 + FE_1 - 2EF_1 = 0$

(c) $GC_1 + FG_2 - 2GF_2 = 0$

(d) $EE_2 - FE_1 + 2EF_1 = 0$

8. A necessary and sufficient condition for a curve $u = u(t)$, $v = v(t)$ on a surface $r = r(u, v)$ to be geodesic is that

(a) $U\frac{\partial T}{\partial \dot{v}} + V\frac{\partial T}{\partial \dot{u}} = 0$

(b) $U\frac{\partial T}{\partial v} - V\frac{\partial T}{\partial u} = 0$

(c) $U\frac{\partial T}{\partial \dot{v}} - V\frac{\partial T}{\partial \dot{u}} = 0$

(d) $U - V\frac{\partial T}{\partial \dot{u}} = 0$

9. The second fundamental form is

(a) $Edu^2 + 2Fdudv + Gdv^2$

(b) $Ldu^2 + 2Mdudv + Ndv^2$

(c) $Lh^2 + Mhk + N$

(d) $Ldv^2 + 2Mdu + Ndu^2$

10. The Gaussian Curvature K is defined by

(a) $K = \frac{1}{2}(K_a + K_b)$ (b) $K = K_a K_b$

(c) $K = \frac{1}{2}\sqrt{K_a K_b}$ (d) $K = LN - M^2$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $[r', r'', r'''] = \kappa^2 \tau$.

Or

(b) Find the curvature and the torsion of the curve given by $r = \{a(3u - u^3), 3au^2, a(3u + u^3)\}$.

12. (a) Prove that the osculating plane at any point P has three point contact with the curve at P.

Or

(b) Prove that the projection C_1 of a general helix C on a plane perpendicular to its axis has its principal normal parallel to the corresponding principal normal of the helix and its corresponding curvature is given be $k = k_1 \sin^2 \alpha$.

13. (a) Show that the matrix is a positive definite quadratic form in du, dv.

Or

(b) Find E,F,G and H for the paraboloid

$x = y, y = v, z = u^2 - v^2$

14. (a) Prove that, on the general surface, a necessary and sufficient condition that the curve $v = c$ be a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$ when $v = c$ for all values of u.

Or

(b) On the paraboloid $x^2 - y^2 = z$ find the orthogonal trajectories of the sections by the planes z = constant.

15. (a) Prove that if the orthogonal trajectories of the curves v = constant are geodesics, then H^2 / E is independent of u.

Or

(b) If K is the normal curvature in a direction making an angle ψ write the principle direction v = constant then prove that $K = K_a \cos^2 \psi + K_b \sin^2 \psi$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Serret Frenet formulae.

Or

(b) Show that the length of the common perpendicular d of the tangents at two near points distance s apart is approximately given by $d = \frac{k \tau s^3}{12}$.

17. (a) Define the osculating sphere and the centre A spherical curvature. If a curve lies on a sphere show that ρ and σ are related by $\frac{d}{ds}(\sigma \rho') + \frac{\rho}{\sigma} = 0$.

Or

(b) Prove that $\frac{K}{\tau}$ = constant is a characteristic property of helices.

18. (a) Find E, F, G and H for the anchor ring and find the area of the anchor ring corresponding to the domain $0 \le u \le 2\pi$ and $0 \le v \le 2\pi$.

Or

(b) If (l', m') are the direction coefficients of a line which makes an angle $\pi/2$ with the line whose direction coefficients are (l, m), then prove that $l' = -\frac{1}{H}(Fl + Gm), m' = \frac{1}{H}(El + Fm)$.

19. (a) Prove that every helix on an cylinder is a geodesic.

Or

(b) Prove that any curve $u = u(t), v = v(t)$ on a surface $r = r(u, v)$ is a geodesic if and only if the principal normal at every point on the curve is normal to the surface.

20. (a) State and prove Liouville's formula for Kg.

Or

(b) Prove that a necessary and sufficient condition for a curve on a surface to be a line of curvature is $kdr + dN = 0$ at each point on the line of curvature where K is the normal curvature in the direction dr of the line of curvature.

-es) **Reg. No. :** ....

ue No. : 5674 **Sub. Code : ZMAM 25**

(CBCS) DEGREE EXAMINATION, APRIL 2022

Second Semester

Mathematics — Core

.SEARCH METHODOLOGY AND STATISTICS

For those who joined in July 2021 onwards)

Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

hoose the correct answer :

hich one of the following is not true

) the title page of a research report should not be numbered

) the pages of the main body of the text are numbered with Arabic numerals

the page of preliminary sections should be numbered using Roman numerals

the page of preliminary sections should be numbered using Arabic numerals

2. Typically, abstracts are between ———— and ———— words in length.

(a) 10 and 20 (b) 250 and 300

(c) 1200 and 1500 (d) 5 and 10

3. Let the joint p.d.f of X_1 and X_2 be $f(x_1, x_2) = \frac{x_1 + x_2}{21}$, $x_1 = 1,2,3$, $x_2 = 1,2$

$= 0$, elsewhere

Then $P_r(X_1 = 3)$ is

(a) $\frac{3}{7}$ (b) $\frac{4}{21}$

(c) $\frac{5}{21}$ (d) $\frac{1}{7}$

4. The m.g.f. $M(t_1, t_2)$ of the joint distribution of X and Y is

(a) $E(t_1X + t_2Y)$ (b) $E(e^{t_1X \cdot t_2Y})$

(c) $E(e^{t_1X + t_2Y})$ (d) $E(t_1X + t_1Y)$

5. If $(1-2t)^{-6}$, $t < \frac{1}{2}$ is the m.g.f. the random variable X, then X is

(a) $\chi^2(6)$ (b) $\chi^2(3)$

(c) $\chi^2(-6)$ (d) $\chi^2(12)$

r a gamma distribution σ^2 is

$\alpha\beta^2$ (b) $\alpha\beta$

$\alpha^2\beta$ (d) $\frac{\alpha}{\beta^2}$

$x_1 = 2y_1 + y_2$, $x_2 = y_2$ then the value of J is

–2 (b) 2

1/2 (d) –1/2

W is $n(0,1)$ and V is $\chi^2(r)$ and if W and V stochastically independent, the which one of following is at distribution

$\frac{W}{\sqrt{V/r}}$ (b) $\frac{W}{\sqrt{Vr}}$

$\frac{W}{(V/r)}$ (d) $\sqrt{\frac{W}{V/r}}$

9. Let X_1 and X_2 be stochastically independent with normal distribution $n(\mu_1, \sigma_1^2)$ and $n(\mu_2, \sigma_2^2)$ respectively. Then $Y = X_1 - X_2$ is

(a) $n(\mu_1 - \mu_2, \sigma_1^2 - \sigma_2^2)$

(b) $n(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$

(c) $n(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$

(d) $n(\mu_1\mu_2, \sigma_1^2 . \sigma_2^2)$

10. If X_1, X_2, X_n denote a random sample from a distribution with m.g.f. $M(t)$. Then m.g.f. of $\sum_1^n \frac{X_i}{n}$ is

(a) $M(t)^n$ (b) $M\left(\frac{t}{n}\right)^n$

(c) $M(t^n)$ (d) $M(tn)^n$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Write an abstract for your research project (you can choose your own topic).

Or

(b) What is methodology and why is it important?

12. (a) If the random variable X_1 and X_2 have the joint p.d.θ $f(x_1, x_2) = 2e^{-x_1 - x_2}$, $0 < x_1 < x_2$, $0 < x_2 < \infty$ and zero elsewhere, prove that X_1 and X_2 are stochastically dependent.

Or

(b) Let X_1 and X_2 have the joint p.d.f. $f(x_1, x_2) = 2$, $0 < x_1 < x_2 < 1$, zero elsewhere. Find the marginal probability density functions and the conditional p.d.f. of X_1 given $X_2 = x_2$, $0 < x_2 < 1$.

13. (a) Let X have a gamma distribution with $\alpha = r/2$, when r is a positive integer and $\beta > 0$. Define $Y = 2X/\beta$. Find the p.d.f. of Y.

Or

(b) If the random variable X is $N(\mu, \sigma^2)$, $\sigma^2 > 0$, prove that the random variable $V = \frac{(X-\mu)^2}{\sigma^2}$ is $\chi^2(1)$.

14. (a) Let X have the p.d.f. $f(x) = 1$, $0 < x < 1$, zero elsewhere. Show that the random variable $Y = -2 \log X$ has a chi-square distribution with 2 degree of freedom.

Or

(b) Let X have the p.d.f. $f(x) = x^2/9$, $0 < x < 3$, zero elsewhere. Find the p.d.f. of $Y = X^3$.

15. (a) Let X_1 and X_2 be stochastically independent with normal distribution $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively. Let $Y = X_1 - X_2$. Using m.g.f. technique, find the p.d.f. of Y.

Or

(b) Let $\overline{X}$ denote the mean of a random sample of size 128 from a gamma distribution with $\alpha = 2$ and $\beta = 4$. Approximate $\Pr(7 < \overline{X} < 9)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) (i) Explain the importance of Literature review.

(ii) Write a short note on plagiarism.

Or

(b) What are the different components of a research project? Explain your answer.

17. (a) Show that $E[E(X_2 \mid X_1)] = E(X_2)$ and $\text{var}[E(X_2 \mid X_1) \le \text{var}\, X_2$.

Or

(b) Show that X_1 and X_2 are independent if and only if $M(t_1, t_2) = M(t_1, 0)\, M(0, t_2)$.

18. (a) Define a gamma distribution and obtain its m.g.f. mean and variance.

Or

(b) Compute the measures of Skewness and Kurtosis of a gamma distribution with parameters α and β.

19. (a) Let X_1, X_2, X_3 denote a random sample of size 3 from a standard normal distribution. Let Y all note the statistic that is the sum of the squares of the sample observations. Find the p.d.f. of Y.

Or

(b) Derive a t-distribution.

20. (a) Let X_i denote a random variable with mean μ_i and variance σ_i^2, $i = 1, 2, .. n$. Let $X_1, X_2, .., X_n$ be independent and let $k_1, k_2, ... k_n$ denote real constants. Compute the mean and variance of $Y = k_1 X_1 + k_2 X_2 + ... + k_n X_n$.

Or

(b) State and prove the central limit theorem.

———

(7 pages) **Reg. No. :** ....

Code No. : 6373 **Sub. Code : ZMAM 25**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022

Second Semester

Mathematics – Core

RESEARCH METHODOLOGY AND STATISTICS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The one person who will almost certainly feature in the acknowledgement is

   (a) your father (b) your supervisor

   (c) your principal (d) your friend

2. Typically, abstracts are between ———— and ———— words in length.

   (a) 250 and 300 (b) 25 and 30

   (c) 2020 and 3000 (d) 5 and 10

3. Suppose a box contains 3 white balls and 2 black balls two balls are to be drawn successively at random and without replacement the probability that both balls drawn are black in

   (a) $\frac{1}{5}$ (b) $\frac{1}{10}$

   (c) $\frac{2}{5}$ (d) $\frac{1}{4}$

4. Let the joint p.d.f of X_1 and X_2 be

$$f(x_1,x_2)=\begin{cases}x_1+x_2, & 0<x_1<1, 0<x_2<1\\ 0 & elsewhere\end{cases}$$

   The marginal probability density function $f(x_1)$ in $0<x_1<1$ is

   (a) x_1+x_2 (b) x_1+1

   (c) $x_1 \times x_2$ (d) x_1

5. If X has the p.d.f $f(x)=\begin{cases}\frac{1}{4}xe^{-x/2} & 0<x<\infty\\ 0 & elsewhere\end{cases}$, then X is

   (a) $\chi^2(2)$ (b) $\chi^2(8)$

   (c) $\chi^2(-\frac{1}{2})$ (d) $\chi^2(4)$

6. If $M(t) = \rho^{3t+8t^2}$, then σ is

(a) 4 (b) 8

(c) 3 (d) 16

7. If $x_1 = \frac{1}{2}(y_1 + y_2)$, $x_2 = \frac{1}{2}(y_1 - y_2)$ then the value of J is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) 2 (d) –2

8. If U and V are stochastically independent chi-square variables with r_1 and r_2 degrees of freedom, then which one of the following is an F-distribution

(a) $\frac{Ur_1}{Vr_2}$ (b) $\frac{U/r_1}{V/r_2}$

(c) $\sqrt{\frac{U/r_1}{V/r_2}}$ (d) $\frac{U/r_2}{V/r_1}$.

9. Let X_1 and X_2 be stochastically independent with normal distribution $n(6,1)$ and $n(7,1)$ respectively, then $X_1 - X_2$ is

(a) $n(-1,2)$ (b) $n(1,2)$

(c) $n(-1,1)$ (d) $n(0,1)$

10. Let $\overline{X}$ denote the mean of a random sample of size 128 from a gamma distribution with $\alpha = 2$ and $s = 4$ the variance of $\overline{X}$ is

(a) $\frac{8}{128}$ (b) $\frac{3}{128}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Write a model Acknowledgement for your research project.

Or

(b) Why do we have a literature review?

12. (a) Show that the random variables X_1 and X_2 with joint p.d.f $f(x_1, x_2) = 12x_1x_2(1 - x_2)$, $0 < x_1 < 1, 0 < x_2 < 1$, zero elsewhere, are stochastically independent.

Or

(b) Two dimensional random variable (x, y) has the joint p.d.f. $f(x, y) = 8xy, 0 < x < y < 1$, zero elsewhere. Find marginal and conditional distributions.

13. (a) Find the m.g.f of a gamma distribution.

Or

(b) If the random variable x is $N(\mu,\sigma^2)$ $\sigma^2 > 0$, prove that the random variable $W = (x-\mu)/\sigma$ is $N(0,1)$.

14. (a) Show that $S^2 = \frac{1}{n}\sum_1^n (x_i - \bar{x})^2 = \frac{1}{n}\sum_1^n x_1^2 - \bar{x}^2$, what $\bar{X} = \sum_1^n X_i / n$.

Or

(b) Let X have the p.d.f. $f(x) = 1$, $0 < x < 1$ zero elsewhere. Show that the random variable $y = -2\log x$ has a chi-square distribution with z degrees of freedom.

15. (a) If $X_1, X_2, \ldots X_n$ is a random sample from a distribution with m.g.f $M(t)$, show that the m.g.f of $\sum_1^n x_i$ and $\sum_1^n X_i / n$ are respectively, $[M(t)]^n$ and $[M(t/n)]^n$.

Or

(b) Let $\bar{X}$ denote the mean of a random sample of size 100 from a distribution test is $\chi^2(50)$. Compute an approximare value of $\Pr(49 < \bar{X} < 51)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) What are the different components of a research project? Explain.

Or

(b) (i) Explain how referencing convention are applied according to their sources.

(ii) Explain plagiarism. How will you avoid it in your project report.

17. (a) Let X_1 and X_2 have joint p.d.f $f(x_1, x_2) = 2, 0 < x_1 < x_2 < 1$ zero elsewhere

Find (i) the marginal probability density function

(ii) Conditional p.d.f of x_1 given $x_2 = x_2, 0 < x_2 < 1$

(iii) Condition mean and conditional variance of x_1 given $x_1 = x_2$.

Or

(b) Let $F(x_1, x_2) = 2(x_1^2 x_2^3, 0 < x_1 < x_2 < 1$, zero elsewhere, be the joint p.d.f pg x_1 and x_2 find the conditional mean and variance of x_1. given $x_2 = x_2$, $0 < x_2 < 1$.

18. (a) Find the m.g.f of the normal distribution and hence find the mean and variance of a normal distribution

Or

(b) If the random variable X is $N(\mu,\sigma^2),\sigma^2 > 0$, prove that the random variable $V = (x - \mu^2 / \sigma^2 \, is \, \chi^2(1)$.

19. (a) Let X_1, X_2 be a random sample of size $n = 2$ from a standard normal distribution. Show that the marginal p.d.f of $y_1 = x_1 / x_2$ is that of a Cauchy distribution. You mat take $y_2 = x_2$.

Or

(b) Derive the F-distribution.

20. (a) Let $X_1, X_2, \ldots X_n$ denote a random sample of size $n \geq 2$ from a distribution that is $N(\mu,\sigma^2)$. Let S^2 be the variance of this random sample prove that $\frac{nS^2}{\sigma^2}$ is a χ^2-variable with parameter $n-1$.

Or

(b) State and prove a special case of the central limit theorem.

(8 pages)

Reg. No. : ....

Code No. : 6377 **Sub. Code : ZMAM 31**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Third Semester

Mathematics — Core

ADVANCED ALGEBRA — I

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If dim V=5 then dim Hom (V, V) + dim Hom (V, F) is

(a) 50 (b) 25

(c) 10 (d) 30

2. An R-module M is said to be cyclic if there is an element $m_0 \in m$ such that every $m \in M$ is of the form

(a) $m = rm_o$ for some $r \in R$

(b) $m = m_o{}^n$ for some integer n

(c) $m = r + m_o$ for some $r \in R$

(d) $m = rm_o$ for some $n \in M$

3. If V is finite dimensional over F and if $T \in A(V)$ is singular, then there exists an $S \neq 0$ in A(V) such that

(a) ST=TS=1

(b) ST=TS=0

(c) $vS = 0$ for some $v \neq 0$ in V

(d) ST=TS

4. If $vT = \lambda v$ the vT^k is

(a) $(\lambda v)^k$ (b) λv^k

(c) $\lambda^k v^k$ (d) $\lambda^k v$

5. If M, of dimension m, is cyclic w.r.t. T, then the dimension of MT^k is

(a) $\frac{m}{k}$ (b) $m+k$

(c) $m-k$ (d) m^k

6. Which one of the following is a Jordan block

(a) $\begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 3 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix}$ (d) $\begin{pmatrix} 3 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix}$

7. $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{pmatrix}$ is the companion matrix of

(a) $1+3x+3x^2$

(b) $1+3x+3x^2+x^3$

(c) $-1-3x-3x^2-x^3$

(d) $-1-3x-3x^2+x^3+x^4$

8. Which one of the following is not true for all $A, B \in Fn$

(a) $(A')' = A$

(b) $(A+B)' = B'+A'$

(c) $(AB)' = A'B'$

(d) $(\lambda A')' = \lambda A'$ where $\lambda \in F$

9. The normal transformation N is unitary if and only if its characteristic roots are

(a) real

(b) complex number

(c) all of absolute value 1

(d) all equal to 1

10. The signature of the real quadratic form $x_1{}^2 + 2x_1x_2 + x^2$ is

(a) 0 (b) −1

(c) 1 (d) 2

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define Ham(V, W). Introduce an addition and a scalar multiplication in Hom (V, W). Show that if $S, T \in Hom(V, W)$ than $S + T \in Hom(V, W)$.

Or

(b) Let V be the set of all continuous complex - valued functions on [0, 1]. If $f(t), g(t) \in V$, define $(f|t|, g|t|) = \int f(t)\overline{g|t|}\,dt$. Prove that this defines an inner product on V.

12. (a) If $\lambda \in F$ is a characteristic root of $T \in A(V)$. Prove that for any polynomial of $q(x) \in F(x)$, $q(\lambda)$ is a characteristic root of q(T).

Or

(b) Let V be the vector space of all polynomials over F of degree 3 or less and let D be the differentiation operator defined on V. Find the matrix of D w.r.t. the basis

(i) $1, x, x^2, x^3$

(ii) $1, 1+x, 1+x^2, 1+x^3$

13. (a) If $W \subset V$ is invariant under T, prove that T induces a linear transformation $\overline{T}$ on V/W defined by $(V+W)\overline{T} = vT + w$.

Or

(b) If $T \in A(V)$ is nilpotent, prove that $\alpha_0 + \alpha_1 + T + ... + amT^m$, where the $\alpha_i \in F$, is invertible if $\alpha_o \neq 0$.

14. (a) If V is cyclic relative to T and if the minimal polynonomial of T in F[x] is $p(x)$, then prove that for some basis of V, the matrix of T is C(p(x)), the companion matrix of $p(x)$.

Or

(b) If A is invarible, prove that det $A \neq 0$, det $(A^{-1}) = (\det A)^{-1}$ and det $(ABA^{-1}) = $ det B for all B.

15. (a) If $T \in A(V)$, prove that $T^* \in A(V)$ and $(T^*)^* = T$.

Or

(b) Let N be a normal transformation and suppose that λ and μ are two distinct characteristic roots of N. If V, W are in V are such that $VN = \lambda v, wN = \mu w$ prove that $(v, w) = 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If V and W are of dimensions m and n respectively exhibit a basis of Han (v, w) over F consisting of mn elements.

Or

(b) Let R be a Euclidean ring. Prove that any finitely generated R-module M is the direct sum of a finite number of cyclic submodules.

17. (a) For arbitrary algebras A with unit element over a field F, state and prove the analog of cayley's theorem for groups.

Or

(b) What relation, if any, must exist between characteristic vectors of T belonging to different characteristic roots? Explain your answer.

18. (a) If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, prove that T statisfies a polynomial of degree n over F.

Or

(b) Let $T \in A(v)$ and suppose that $p(x) = q_1(x)^{l1} q_2(x)^{l2} ... q_k(n)^{lk}$ in $F(x)$ is the minimal polynomial of T over F. For each $i = 1,2......k$, define $V_i = \{v \in V \mid (V_{qi}(T))^{li} = 0\}$ prove that $v_i \neq 0$ for each i and $V = V_1 \oplus V_2 \oplus ... \oplus Vk$.

19. (a) Prove that the elements S and T in A(V) are similar in A(V) if and only if they have the same elementary divisions.

Or

(b) For $A, B \in Fn$ and $\lambda \in F$, Prove that

(i) $tr(\lambda A) = \lambda t_r A$

(ii) $tr(A+B) = trA + trB$.

(iii) $tr(AB) = tr(BA)$.

20. (a) Prove that to linear transformation T on V is unitary if and only if it takes an orthonomal basis of V into an orthonormal basis of V.

Or

(b) State and prove that sylvester's law of inertia.

———————

(6 pages) **Reg. No. :** ....

Code No. : 6378 **Sub. Code : ZMAM 32**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Third Semester

Mathematics

GRAPH THEORY

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The number of edges in $K_{4,5}$ is

   (a) 9 (b) 20

   (c) 1 (d) 45

2. Consider the graph G :  In $G-e$, the number of vertices is

   (a) 2 (b) 9

   (c) 6 (d) 8

3. If G is a tree with 16 edges then the number of vertices of G is

   (a) 17

   (b) 15

   (c) 0

   (d) any number between 1 and 16

4. Which one of the following is nit true?

   It e is link of G then

   (a) $\gamma(G \cdot e) = \gamma(G) - 1$

   (b) $\varepsilon(G \cdot e) = \varepsilon(G) - 1$

   (c) $\omega(G \cdot e) = \omega(G) - 1$

   (d) $G \cdot e$ is a tree is G is a tree

5. In the Konigsberg bridge problem, the number of bridge is

   (a) 5 (b) 7

   (c) 9 (d) 11

6. In $C_{2,5}$, the number of edges is

   (a) 7 (b) 10

   (c) 3 (d) 5

7. For a graph G with 12 vertices, it $\alpha = 3$ then β is

   (a) 15 (b) 36

   (c) 9 (d) 3

8. The value of $r(3,3)$ is

   (a) 6 (b) 9

   (c) 0 (d) 3

9. If G is 5-critical then

   (a) $\delta = 5$ (b) $\delta \geq 4$

   (c) $\delta \geq 5$ (d) $\delta \leq 4$

10. There are only ———— types of graph G for which $\chi = \Delta + 1$?

    (a) 2 (b) 3

    (c) 4 (d) 5

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain the following with suitable examples

    (i) Simple graph

    (ii) Spanning subgraph

    (iii) Walk

    (iv) Path

    (v) Cycle.

    Or

    (b) Prove that $\sum_{v \in V} d(v) = 2\varepsilon$ and hence show the number of vertices of odd degree is even in any graph.

12. (a) If G is a tree, prove that $\varepsilon = \gamma - 1$.

    Or

    (b) Prove that a vertex v of a tree G is a cut vertex of G if and only if $d(v) > 1$.

13. (a) Prove that $C(G)$ is well defined.

    Or

    (b) Define a maximum matching and a minimum covering. Let M be a matching k and be a covering such that $|M| = |K|$, then prove that M is a maximum making and K is a minimum covering.

14. (a) If $\delta > 0$, prove that $\alpha' + \beta' = \gamma$.

Or

(b) Prove that $r(k, l) \leq \binom{k+l-2}{k-1}$.

15. (a) If G is k-critical, prove that $\delta \geq k - f$.

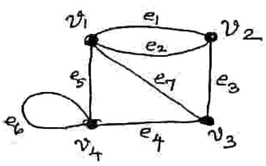
Or

(b) For any graph G, prove that $\prod_k(G)$ is a polynomial is k of degree r, with integer coefficients, leading term k^r and constant term zero and the coefficients of $\prod_k(G)$ alternative in sign.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Define the incidence and adjacency matrices of a graph. Find the two matrices for the following graph.



Or

(b) Obtain a necessary and sufficient condition for graph to be bipartite.

17. (a) Define a cut edge with an example. Prove that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

Or

(b) State and prove Whitney's theorem for 2-connected graphs.

18. (a) Let G be a simple graph with degree sequence $(d_1, d_2, \ldots dr)$ when $d_1 \leq d_2 \leq \ldots \leq d_r$ and $r \geq 3$. Suppose that there is no value of m less than $r/2$ for which $d_m \leq m$ and $d_{r-m} < r - m$. Prove that G is Hamiltonian.

Or

(b) Prove that a matching M in G is a maximum matching if and only if G contains no M-augmenting path.

19. (a) If G is simple, prove either $\chi' = \Delta \theta \chi' = \Delta + 1$.

Or

(b) Prove that $r(k, k) \geq 2^{k/2}$.

20. (a) For any positiver integer k, prove that there exists a k-chromatic graph containing no triangle.

Or

(b) State and prove Brook's theorem.

(7 pages)

Reg. No. : ....

Code No. : 6379 **Sub. Code : ZMAM 33**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Third Semester

Mathematics — Core

MEASURE AND INTEGRATION

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Let A be the set of irrational numbers in [0, 1]. Then $m^*(A)$ is

   (a) ∞ (b) 0

   (c) 1 (d) 1/2

2. If A is a measurable set of finite outer measure that is contained in B then $m^*(B \sim A) = m^*(B) - m^*(A)$. This property is known as

   (a) finite subadditivity (b) translation invariant

   (c) countably additive (d) excision property

3. For a function of defined on E, $f^+(x)$ is defined by

   (a) $\max\{f(x) - f(x)\}$ (b) $\max\{f(x), 0\}$

   (c) $\max\{-f(x), 0\}$ (d) $\min\{f(x), 0\}$

4. If for each $\varepsilon > 0$, then is an index N for which $|f - f_n| < \varepsilon$ on A for all $n \geq N$, we say that

   (a) $\{f_n\}$ converges to f uniformly on A

   (b) $\{f_n\}$ converges to f pointwise a.e. on A

   (c) $\{f_n\}$ converges to f pointwise on A

   (d) $\{f_n\} \to f$ as $n \to \infty$

5. Let f be a nonnegative measurable function on E. then for any $\lambda > 0$, $m\{x \in E / f(x) \geq \lambda\}$ is

   (a) $= \lambda \int_E f$ (b) $\leq \lambda \int_E f$

   (c) $\geq \lambda \int_E f$ (d) $= \frac{1}{\lambda} \int_E f$

6. Define $g_n = \chi_{(n,n+1)}$. Then $\lim_{n\to\infty} \int_{\mathbb{R}} g_n$ is

   (a) 0 (b) 1

   (c) ∞ (d) n

7. $[-5f]^+$ is

   (a) $-5f^+$ (b) $5f^-$

   (c) $5f^+$ (d) $-5f^-$

8. For all $a \leq u \leq v \leq b$, $\int_u^v Diff_h f$ is

   (a) 0 (b) $Av_h f(v) + Av_h f(u)$

   (c) $Av_h f(v) - Av_h f(u)$ (d) $Av_h f(u) - Av_h f(v)$

9. A function f on $[a, b]$ is absolutely continuous on $[a,b]$ if and only if

   (a) it is an indefinite integral over $[a,a]$

   (b) $\int_a^b f' = f(a) - f(a)$

   (c) f is differentiable almost everywhere on (a, a)

   (d) f is differentiable

10. If $\{A,B\}$ is a Hahn decomposition for r, then r^+ and r^- are defined by

    (a) $r^+(E) = r(E \cap A),\ r^-(E) = -r(E \cap B)$

    (b) $r^+(E) = -r(E \cap A),\ r^-(E) = r(E \cap B)$

    (c) $r^+(E) = r(E \cup A),\ r^-(E) = -r(E \cup B)$

    (d) $r^+(E) = -r(E \cup A),\ r^-(E) = r(E \cup B)$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define a measurable set and prove that any set of outer measure zero is measurable.

    Or

    (b) If m is the Lebesgue measure and if $\{A_K\}_{k=1}^{\infty}$ is an ascending collection of measurable sets, prove that $m\left(\bigcup_{k=1}^{\infty} A_k\right) = \lim_{k\to\infty} m(A_k)$.

12. (a) Let f be an extended real - valued function on E. If f is measurable on E and $f = g$ a.e. on E, prove that g is measurable on E.

    Or

    (b) Let $\{f_n\}$ be a sequence of measurable function on E that converges pointwise a.e. on E to the function f. Prove that f is measurable.

[P.T.O.]

13. (a) Let f be a bounded measurable function on a set of finite measure E. Prove that f is integrable over E.

Or

(b) State and prove Chebychev's inequality.

14. (a) Let f be integrable over E and $\{E_n\}_{n=1}^{\infty}$ a disjoint countable collection of measurable subsets of E whose union is E. Prove that $\int_E f = \sum_{n=1}^{\infty} \int_{E_n} f$.

Or

(b) Define the total variation $TV(f)$ of f on $[a,b]$. If f is a Lipschitz function on $[a,b]$, prove that f is of bounded variation of $[a,b]$.

15. (a) Define absolutely continuous functions. Prove that absolutely continuous functions are continuous. Is the converse true? Justify.

Or

(b) State and prove the Jordan decomposition theorem.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) Prove that the outer measure of an interval is its length.

Or

(b) (i) State and prove that Borel - Cantelli lemma.

(ii) State the continuity properties of Lebesque measure.

17. (a) Let f and g be measurable functions on E that are finite a.e. on E. For any α and β, prove that $\alpha f + \beta g$ is measurable on E and that fg is measurable on E.

Or

(b) State and prove Egoroff's theorem.

18. (a) State and prove the bounded convergence theorem.

Or

(b) Let f and g be nonnegative measurable functions on E. For any $\alpha > 0$ and $\beta > 0$, prove that $\int_E (\alpha f + \beta g) = \alpha \int_E f + \beta \int_E g$.

19. (a) State and prove the Lebesgue dominated convergence theorem.

Or

(b) State and prove the Vitali covering lemma.

20. (a) Let the function f be absolutely continuous on $[a,b]$. Prove that f is the difference of increasing absolutely continuous functions and in particular, is of bounded variation.

Or

(b) Let γ be a signed measure on the measurable space (X,M) and E a measurable set for which $0 < \gamma(E) < \infty$. Prove that there is a measurable subset A of E that is positive and of positive measure.

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(8 pages)

Reg. No. : ....

Code No. : 6380 **Sub. Code : ZMAM 34**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Third Semester

Mathematics — Core

TOPOLOGY — I

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Let $X = \{a,b,c\}$, $\tau_1 = \{\phi, X, \{a\}, \{a,ab\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{b,cb\}\}$. The largest topology contained in τ_1 and τ_2 is

   (a) $\{\phi, X\}$

   (b) $\{\phi, X, \{ab\}\}$

   (c) $\{\phi, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}\}$

   (d) the discrete topology on X

2. Let $Y = (0,1]$ be a subspace of R. Let $A = \left(0, \frac{1}{2}\right)$ the closure of A in Y is

   (a) $(0, 1/2]$ (b) $[0, 1/2]$

   (c) $(0, 1/2)$ (d) $(0, 1]$

3. Let $f : \{a,b,c\} \to \{1,2,3\}$ be defined by $f(a) = 2, f(b) = 3$, $f(c) = 3$. Let $V = \{1,3\}$, then $f^{-1}(V)$ is

   (a) $\{a,b\}$ (b) $\{b,c\}$

   (c) $\{a,c\}$ (d) $\{a,b,c\}$

4. Consider the identity functions $f : R \to R_l$ and $g : R_l \to R$, then

   (a) both f and g are continuous

   (b) f is continuous but g is not continuous

   (c) f is not continuous but g is continuous

   (d) neither f nor g is continuous

5. In R^n, we have $d(x,y) \le \sqrt{n}\rho(x,y)$, this inequality shows that

(a) $B_\rho(x, \varepsilon/\sqrt{n}) \subset B_d(x,\varepsilon)$

(b) $B_\rho(x,\varepsilon) \subset B_d\left(x, \frac{\varepsilon}{\sqrt{n}}\right)$

(c) $B_\rho(x,\varepsilon) \subset B_d(x,\varepsilon)$

(d) $B_d(x,\varepsilon) \subset B_\rho\left(x, \frac{\varepsilon}{\sqrt{n}}\right)$

6. The metric that induces the product topology on R^ω is

(a) $D(x,y) = \sup\{\bar{d}(x_i, y_i)\}$

(b) $D(x,y) = \sup\{|x_n - y_n|\}$

(c) $D(x,y) = \sup\left\{\frac{\bar{d}(x_i,y_i)}{i}\right\}$

(d) $D(x,y) = \sup\left\{\frac{i}{\bar{d}(x_i,y_i)}\right\}$

7. If X is connected then

(a) ϕ is the only subset which is both open and closed

(b) X is the only subset which is both open and closed

(c) ϕ and X are the only subsets which are both open and closed

(d) we can find a subset $A(\neq \phi, X)$ which is both open and closed

8. Which one of the following is compact in R

(a) [0, 1] (b) [0, 1)

(c) (0, 1) (d) (0, 1]

9. Which one of the following is NOT a subsequence of (x_n)

(a) $\{x_5, x_6, x_7, \ldots.\}$

(b) $\{x_3, x_6, x_9, \ldots.\}$

(c) $\{x_2, x_1, x_3, x_2, x_4, x_3 \ldots.\}$

(d) $\{x_{100}, x_{200}, x_{300}, \ldots\}$

10. Which one of the following is not locally compact

(a) R

(b) Q

(c) R^n

(d) Every simply ordered set having the *l.u.b.* property

[P.T.O.]

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define a basis for a topology on X. If B is a basis for a topology τ on X, prove that τ equals the collection of all unions of elements AB.

Or

(b) Let Y be a subspace of X. Let A be a subset of Y. Let $\overline{A}$ denote the closure of A in X. Prove that the closure of A in Y equals $\overline{A} \cap Y$.

12. (a) If $\mathcal{B}$ is a basis for the topology on X and $\mathcal{C}$ is a basis for the topology on Y, prove that $\mathcal{D} = \{B \times C / B \in \mathcal{B}, c \in \mathcal{C}\}$ is a basis for the topology of $X \times Y$.

Or

(b) State and prove the pasting lemma.

13. (a) Let X be a metric space with metric d. Define $\overline{d} : X \times x \to R$ by $\overline{d}(x,y) = \min\{d(x,y),1\}$. Prove that $\overline{d}$ is a metric that induces the same topology as d.

Or

(b) State and prove the sequence lemma.

14. (a) Prove that the union of a collection of connected subspace of X that have a point in common is connected.

Or

(b) Prove that every closed subspace of a compact space is compact.

15. (a) Prove that compactness implies limit point compactness.

Or

(b) Let X be a Hausdorff space. Prove that X is locally compact if and only if given x in X, and given a neighborhood U of x, there is a neighborhood V of x such that $\overline{V}$ is compact and $\overline{V} \subseteq U$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) Define the standard topology, the lower limit topology and the K - topology on R. Find the relation between these topologies.

Or

(b) Let A be a subset of the topological space X. Let A' be the set of all limit points of A. Prove that $\overline{A} = A \cup A'$ and hence show that A is closed if and only if it contains all its limit points.

17. (a) Let X and Y be topological space; let $f : X \to Y$. Prove that the following are equivalent

(i) f is continuous

(ii) for every subset A of X, one has $f(\overline{A}) \subseteq \overline{f(A)}$

(iii) for every closed set B of Y, the set $f^{-1}(B)$ is closed in X.

Or

(b) Let $f : A \to \pi_{\alpha \in J} X_\alpha$ be given by the equation $f(a) = (f_\alpha(a))_{\alpha \in J}$, where $f_\alpha : A \to X_\alpha$ for each α. Let πX_α have the product topology. Prove that the function f is continuous if and only if each function f_α is continuous.

18. (a) Prove that the topologies on R^n induced by the Euclidean metric d and the square metric ρ are the same as the product topology on R^n.

Or

(b) State and prove the uniform limit theorem.

19. (a) Prove that a finite Cartesian product of connected spaces is connected.

Or

(b) State and prove the tube lemma.

20. (a) Let X be a metrizable space. If X is sequentially compact, prove that X is compact.

Or

(b) Let X be a space. If X is locally compact Hausdorff prove that there exists a space Y satisfying

(i) X is a subspace of Y

(ii) $Y - X$ consists of a single point

(iii) Y is a compact Hausdorff space.
